Optimization for Dynamic Transportation Systems via the Internal Model Principle

KTH - DCS & Digital Futures seminar 22.05.2025

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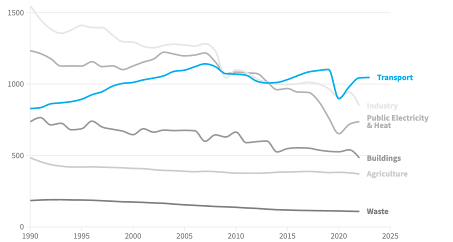
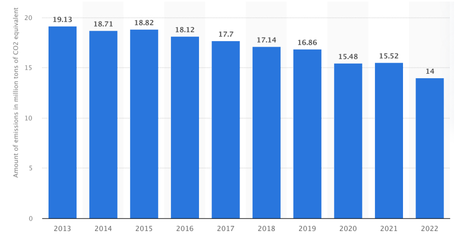
Harmful consequences of transportation

Transportation is the largest source of emissions in the EU



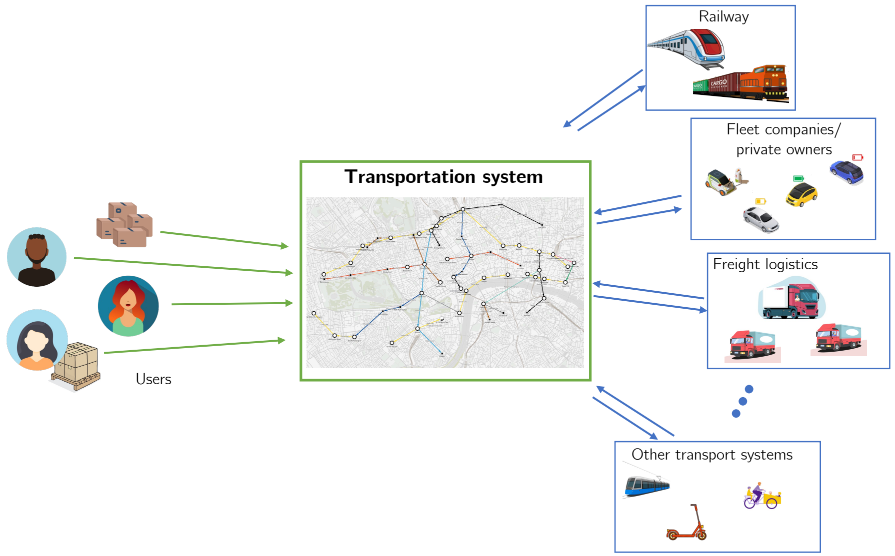
Greenhouse emissions (MtCO2e) Source: UNFCCC

The transport sector in Sweden emits as much greenhouse gas as 1% of the Amazon rainforest can absorb



Source: Statista

A network of networks



Emerging features

- Sustainability and transition
- Increasing complexity and heterogeneity
- Faster dynamics
- Complex models and problems
- Resilience and security

EVs

Micro vehicles

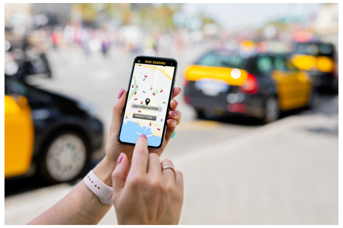
Multi-modal transit

Ridesharing

Emerging features

- Sustainability and transition
- Increasing complexity and heterogeneity
- Faster dynamics
- Complex models and problems
- Resilience and security

EV vs combustion

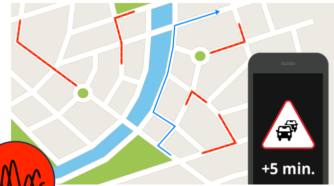

Users-in-the-loop


Connected vehicles

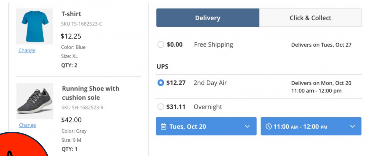
New features = new challenges

All these new features make it difficult to operate the system optimally

Dynamic factors:
- Multi-modal routes
- User adaptation
- Transport demands
- …



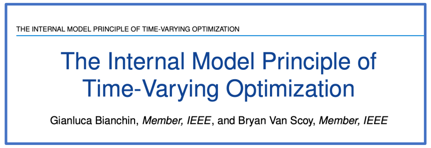
Dynamic factors:
- Response to congestion
- Transport demands
- Vehicle type
- Tolls
- …



Dynamic factors:
- Cargo loads
- Vehicle type
- Cargo modes
- Seasonal demand
- …

Talk outline

1 – Making dynamic decisions


THE INTERNAL MODEL PRINCIPLE OF TIME-VARYING OPTIMIZATION

The Internal Model Principle of Time-Varying Optimization

Gianluca Bianchin, *Member, IEEE*, and Bryan Van Scoy, *Member, IEEE*

2 - Making dynamic decisions in dynamic (!) environments

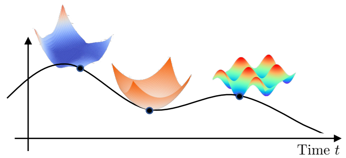
3 - Conclusions

Decision-making in dynamic environments

General optimization problem:

$$\min_{x \in \mathbb{R}^n} f(x, \theta(t))$$

- $\theta(t) \in \Theta \subseteq \mathbb{R}^p$ $\rightarrow$ time-varying parameter vector
- $f(x, \theta)$ $\rightarrow$ smooth convex function to be minimized



Temporal variability satisfies:

$$\dot{\theta}(t) = s(\theta(t))$$

Objective: At each time t, determine an optimal decision $x^\star(t)$

Gradient-type methods

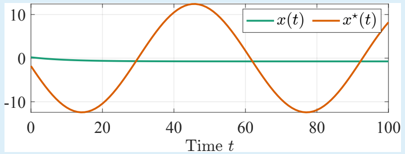
Gradient-type algorithms:

Algorithm has access to gradient oracles:

$$(t, x) \mapsto \nabla_x f(x, \theta(t))$$

Basic gradient-flow

$$\dot{x}(t) = -\eta \nabla_x f(x(t), \theta(t))$$

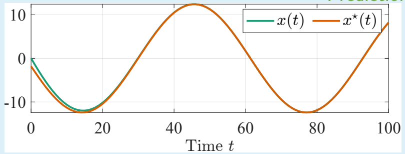


Computes an **approximate** optimizer

[Hazan 2016, Absil 2006, Hall 2015, ...]

Prediction-correction

$$\dot{x}(t) = -H^{-1}[\eta \overbrace{\boxed{\nabla_x f(x(t), \theta(t))}}^{\text{Correction}} + \underbrace{\boxed{\nabla_{x\theta} f(x(t), \theta(t)) \cdot s(\theta(t))}}_{\text{Prediction}}]$$



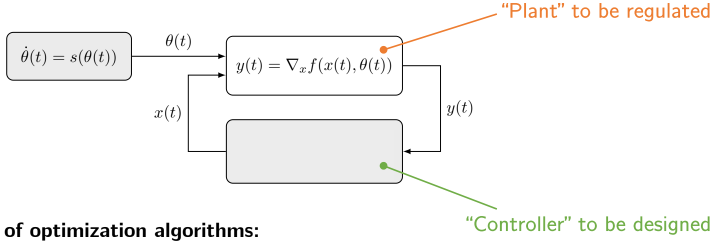
Computes an **exact** optimizer

...but **very strong assumptions:**

(strongly cvx, hessian, sensitivity, $s(\theta)$, ...)

[Zhao 1998, Fazylab 2017, Raveendran 2022, ...]

Optimization algorithms as feedback controllers



General class of optimization algorithms:

$$\dot{z}(t) = F_c(z(t), y(t))$$
$$x(t) = G_c(z(t))$$

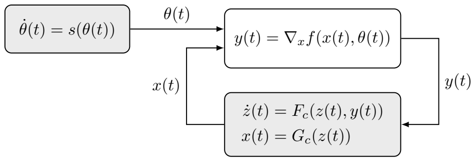
Problem 1 (Algorithm design):

Design $F_c(z, y)$ and $G_c(z)$ such that $y(t) \to 0$ as $t \to \infty$

Problem 0 (Minimal knowledge):

What is the "minimal knowledge" required to design an algorithm from the class achieving exact asymptotic tracking?

Minimal knowledge: considerations



Temporal-variability properties:

(E1) $\theta(t)$ is measurable

[Hazan 2016, Hall 2015, ...]

(E2) $s(\theta)$

[Bastianello 2024]

(E3) $\theta(t)$ and $s(\theta)$ are known

[Zhao 1998, Fazylab 2017, Raveendran 2022, ...]

....

Optimization properties:

(O1) Oracle gradient evaluations:

$$(t, x) \mapsto \nabla_x f(x, \theta(t))$$

[Hazan 2016, Hall 2015, ...]

(O2) Gradient $\nabla_x f(x, \theta)$

(O3) Sensitivity $\nabla_{x\theta} f(x, \theta)$

(O4) Hessian $\nabla^2_{xx} f(x, \theta)$

[Zhao 1998, Fazylab 2017, Raveendran 2022, ...]

(O5) Loss is quadratic

[Bastianello 2024, ...]

...

Talk outline

1 – Making dynamic decisions

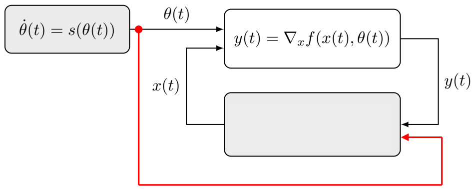
but the variability is measurable

and the variability is unmeasurable

2 - Making dynamic decisions in dynamic (!) environments

3 - Conclusions

The parameter feedback problem



Static optimization algorithm:

$$x(t) = H_c(\theta(t))$$

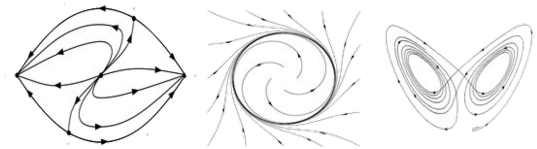
Instrumental notions

Definition

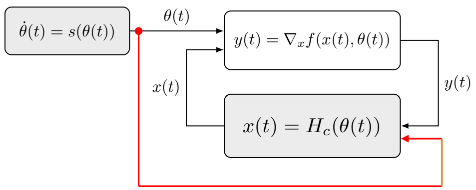
The algorithm **exactly asymptotically tracks an optimizer** if there exists Θ_s, neighborhood of the origin, such that for each $\theta(0) \in \Theta_s$, the solution of the interconnection satisfies $y(t) \to 0$

Definition

- θ_ω is a **limit point** of $\dot\theta(t) = s(\theta(t))$ wrt the initialization $\theta_\circ$ if $\theta(\mathrm{t}_i) \to \theta_\omega$ for some sequence t_i when starting at $\theta_\circ$
- The set of all limit points $\Omega(\Theta_\circ)$ is called **limit set**



The parameter feedback problem (continued)



Static optimization algorithm:

$$x(t) = H_c(\theta(t))$$

Result

The static optimization algorithm achieves exact asymptotic tracking if and only if

$$0 = \nabla_x f(H_c(\theta_\omega), \theta_\omega)$$

at all limit points $\theta_\omega \in \Omega(\Theta_\circ)$

Answer to P0: when $\theta(t)$ is measurable, minimum knowledge required is $\nabla_x f(x, \theta)$

Talk outline

1 – Making dynamic decisions
    but the variability is measurable
    and the variability is unmeasurable

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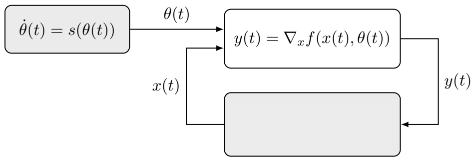
The gradient feedback problem



Dynamic optimization algorithm:

$$\dot z(t) = F_c(z(t), y(t))$$
$$x(t) = G_c(z(t))$$

Assumption:

$\theta(t)$ is exponentially detectable from $y(t)$:

$$\left\|\hat\theta(t) - \theta(t)\right\| \leq M e^{-ct} \left\|\hat\theta(0) - \theta(0)\right\|$$

Result

An algorithm that achieves exact asymptotic tracking **exists** if and only if

$$\boxed{0 = \nabla_x f(H_c(\theta_\omega), \theta_\omega)}$$

for some $H_c(\theta)$ at all limit points $\theta_\omega \in \Omega(\Theta_\circ)$

Static and dynamic algorithms exist under the same conditions!

The internal model principle of optimization

Theorem

The optimization algorithm:

$$\dot z(t) = F_c(z(t), y(t))$$
$$x(t) = G_c(z(t))$$

achieves exact asymptotic tracking if and only if there exists $\sigma(\theta)$:

$$\boxed{\left.\frac{\partial\sigma(\theta)}{\partial\theta}\right|_{\theta=\theta_\omega} s(\theta_\omega) = F_c(\sigma(\theta_\omega), 0),}$$
$$0 = \nabla_x f(G_c(\sigma(\theta_\omega)), \theta_\omega)$$

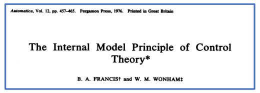
at all limit points $\theta_\omega \in \Omega(\Theta_\circ)$

Implications:

$\theta(t)$ and $z(t)$ must be related, at optimality, by a change of variables:

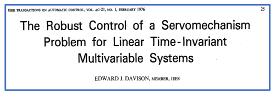
$$z(t) = \sigma(\theta(t))$$

Answer to P0: when $\theta(t)$ is not measurable, minimum knowledge required is $\nabla_x f(x, \theta)$ and $s(\theta)$


Automatica, Vol. 12, pp. 457–465. Pergamon Press, 1976. Printed in Great Britain

The Internal Model Principle of Control Theory*

B. A. FRANCIS† and W. M. WONHAM‡


IEEE TRANSACTIONS ON AUTOMATIC CONTROL, VOL. AC-21, NO. 1, FEBRUARY 1976

The Robust Control of a Servomechanism Problem for Linear Time-Invariant Multivariable Systems

EDWARD J. DAVISON, MEMBER, IEEE

The "internal model principle"

an algorithm can track time-varying optimizers if and only if it incorporates a model of the temporal variability

Implications

Basic gradient-flow

$$\dot x(t) = -\eta \nabla_x f(x(t), \theta(t))$$

satisfies internal model conditions with:

$$s(\theta) = 0, \quad \sigma(\theta) = \text{arbitrarily chosen}$$

Basic gradient flow incorporates an internal model of a constant parameter

Prediction-correction

$$\dot x(t) = -H^{-1}[\eta \nabla_x f(x(t), \theta(t)) + \nabla_{x\theta} f(x(t), \theta(t)) \cdot s(\theta(t))]$$

Prediction-correction asymptotically computes a mapping that zeros the gradient

What if we do not know internal model?

- Fundamental limitation: no exact tracking is possible! (for any algorithm in the class!)
- Robustness-type guarantees can be given (when $\theta(t)$ is bounded):

$$\|y_\infty\| \leq \mathrm{c}\|\theta(0)\|\|S - \hat S\|$$

Optimization design

When $\sigma(\theta) = Id$ (identity operator):

$$z(t) = \theta(t) \qquad s(\theta) = F_c(\theta, 0)$$

Optimization algorithm is a copy of the temporal variability

Algorithm 1: Optimization algorithm design

Data: $s(\theta)$, $\nabla_x f(x, \theta)$, map zeroing gradient $G_c(\theta)$ exponential observer Q, S

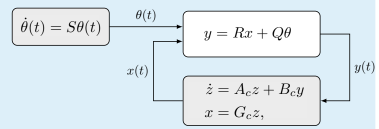
$n_c \leftarrow p$;
$L \leftarrow$ any matrix such that $S - LQ$ is Hurwitz;
$G_c(z) \leftarrow H_c(z)$;
$F_c(z, y) \leftarrow s(z) + L(y - \nabla_x f(H_c(z), z))$;

Result: $F_c(z, y)$, $G_c(z)$, and n_c

Example

Optimization: $\min_{x \in \mathbb{R}^n} f(x(t), \theta(t)) = \frac{1}{2} x(t)^\top R x(t) + x(t)^\top Q \theta(t)$

Temporal variability: $\dot\theta(t) = S\theta(t)$

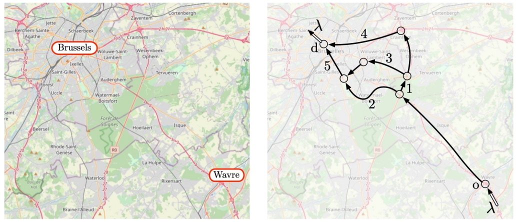


Mapping zeroing the gradient: $H_c(\theta) = -R^\dagger Q \cdot \theta$

Internal model conditions: $\Sigma S = A_c \Sigma, \quad 0 = R G_c \Sigma + Q$

Optimization algorithm: $F_c(z, y) = Sz + Ly, \quad G_c(\theta) = -R^\dagger Q \cdot \theta$

Dynamic traffic assignment



Objective

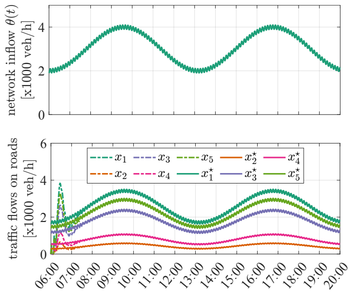
Optimally split traffic demand λ among alternative paths to minimize travel time to destination

- Network state $x \to$ amount of flow routed on each road
- Travel time on road i: $\ell_i(x_i)$
- Travelers minimize their travel time to destination:

$$\min_{x\in\mathbb{R}^{|\mathcal{E}|}} \quad \sum_{i\in\mathcal{E}} \int_0^{x_i} \ell_i(s)ds$$

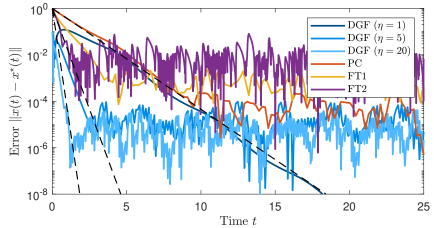
$$\text{subject to:} \quad \sum_{j\in\mathcal{E}:j^-=v} x_j - \sum_{j\in\mathcal{E}:j^+=v} x_j = \delta_v(\lambda(t)), \ \forall v \in \mathcal{V},$$

$$x_i \geq 0, \ \forall i \in \mathcal{E}$$

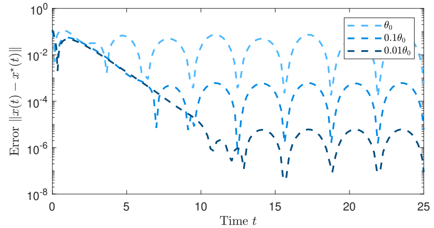


Strengths and caveats

- Convergence can be made arbitrarily fast



- But, in general, it is of local nature



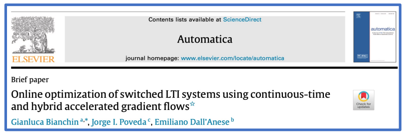
Talk outline

1 – Making dynamic decisions

but the variability is measurable

and the variability is unmeasurable

2 - Making dynamic decisions in dynamic (!) environments


Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Online optimization of switched LTI systems using continuous-time and hybrid accelerated gradient flows

Gianluca Bianchin, Jorge I. Poveda, Emiliano Dall'Anese

Two-point Random Gradient-free Methods for Model-free Feedback Optimization

Amir Mehrnoosh and Gianluca Bianchin

3 - Conclusions

Optimal regulation in dynamic environments

Transportation systems with non-negligible dynamics

$$\dot{x}(t) = Ax(t) + Bu(t) + Ew(t)$$
$$y(t) = Cx(t) + Dw(t)$$

(Stable, controllable, observable)

Optimal output regulation:

$$\min_{u,x,y} \ \psi_t(u,y)$$

Dynamic objectives

$$\text{s.t.} \quad 0 = Ax + Bu + Hw(t)$$
$$y = Cx + Dw(t)$$

Dynamic disturbance

- $\psi_t(x,y) \to$ time-dependent, smooth, strongly convex

Objective: Design a controller so that $(u(t), x(t), y(t)) \to (u^\star(t), x^\star(t), y^\star(t))$

Optimal regulation: related works

$$\begin{aligned} \min_{u,x,y} \quad & \psi_t(u,y) \\ \text{s.t.} \quad & 0 = Ax + Bu + Hw(t) \\ & y = Cx + Dw(t) \end{aligned}$$

- **Output regulation:** track a prescribed reference
  [Davidson 76], [Francis & Wonham 76], [Yoon and Lin 16], ..., [Huang 03,04], [Isidori & Byrnes 90], ...
- **Extremum-seeking:** estimate gradient online
  [Leblanc 22], ... [Wittenmark & Urquhart 95], ... [Krstić & Wang 00], ... , [Feiling et.al. 18]
- **Optimal control (e.g., LQR)**: more general control objective, requires disturbance knowledge
  [Bertsekas 95], ...
- **MPC (real-time/online):** more general control objective, but harder to solve online
  Real-time MPC [Zeilinger et.al. 09], Optimizing control [Garcia & Morari 81], ...
- **Feedback optimization:** this presentation
  [Hauswirth-Bolognani-Hug-Dorfler 20], [Lawrence-Simpson Porco-Mallada 21], [Simpson Porco 22], [Belgioioso et.al. 24], [Carnevale-Mimmo-Notarstefano 24, ...]

Eliminate dependent variables

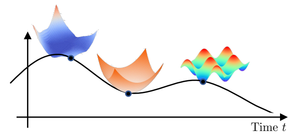
Constrained:

$$\begin{aligned} \min_{u,x,y} \quad & \psi_t(u,y) \\ \text{s.t.} \quad & 0 = Ax + Bu + Hw(t) \\ & y = Cx + Dw(t) \end{aligned}$$

Unconstrained:

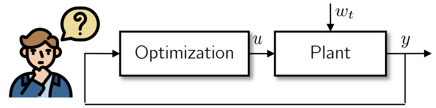
$$\min_{u} \quad \psi_t(u, Gu + Hw(t))$$

$$\min_{x \in \mathbb{R}^n} f(x, \theta(t))$$



Similar form as the first part of this talk!

...now with many additional challenges!



Challenge 1: optimization with feedback

Classical optimization

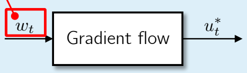
$$\min_{u} \quad \psi_t(u, Gu + Hw(t))$$

- **Basic gradient-flow**:

$$\dot{u} = -\eta \Pi^{\mathsf{T}} \nabla \psi_t(u, Gu + Hw_t)$$

$$\Pi^{\mathsf{T}} = [I, \ G^{\mathsf{T}}]$$

Inapplicable!

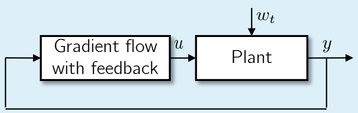


Optimization with plant-in-the-loop:

$$\min_{u} \quad \psi_t(u, Gu + Hw(t))$$

- **Gradient-flow with feedback**:

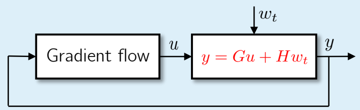
$$\begin{aligned} \dot{x} &= Ax + Bu + Ew_t \\ y &= Cx + Dw_t \\ \dot{u} &= -\eta \Pi^{\mathsf{T}} \nabla \psi_t(u,y) \end{aligned}$$

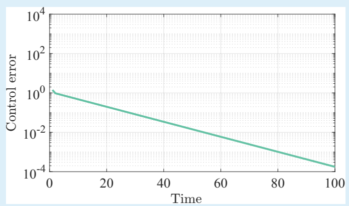


Classical optimization methods are inapplicable!

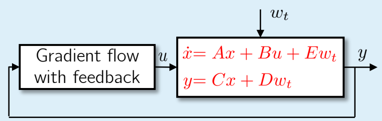
Challenge 2: optimization and plant form a control loop

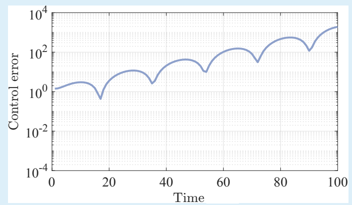
“Static” plant:





“Dynamic” plant:



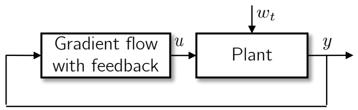


Classical optimization algorithm design fails!

Converge bounds

Gradient-flow with feedback:

$$\dot{x} = Ax + Bu + Ew_t$$
$$y = Cx + Dw_t$$
$$\dot{u} = -\eta \Pi^\mathsf{T} \nabla \psi_t(u, y)$$



Theorem: Suppose

- $\psi_t(\cdot)$ strongly convex
- $\nabla \psi_t(\cdot)$ Lipschitz
- $t \mapsto w_t$ locally absolutely continuous

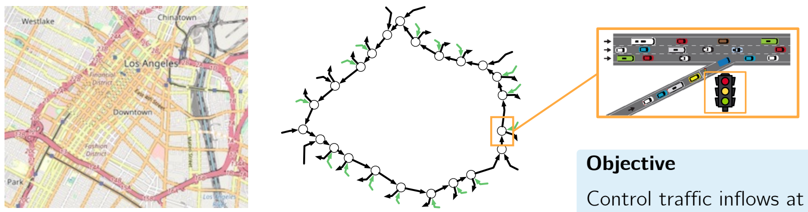
Then, provided that the plant is 'much' faster than the controller ($\eta \leq \bar{\eta}$) :

$$\|(u_t, x_t) - (u_t^*, x_t^*)\| \leq a e^{-bt} \|(u_0, x_0) - (u_0^*, x_0^*)\| + d \operatorname*{ess\,sup}_\tau \|\dot{w}_\tau\| + c \operatorname*{ess\,sup}_\tau \|(\dot{u}_\tau^*, \dot{x}_\tau^*)\|$$

[Bianchin et.al. Automatica, 2022]

Ramp metering

Freeway ramp metering problem



Objective

Control traffic inflows at on-ramps to maximize network throughput

How it is currently done:

- PI-type controller, local at ramps/corridors
- Simulation-based controller tuning, seasonal parameters update

Difficulties:

- Complex (nonlinear) model
- Rapid changes in traffic demand at peak hours

Feedback optimization for ramp metering

Cell Transmission Model:

$$\dot{x}_i = -f_i^{\text{out}}(x) + f_i^{\text{in}}(x)$$
$$f_i^{\text{out}}(x) = \min\{d_i(x_i), \{s_j(x_j)/r_{ij}\}_{j \in i^+}\}$$
$$d_i(x_i) = \min\{\varphi_i x_i, d_i^{\max}\}$$
$$s_i(x_i) = \min\{\beta_i(x_i^{\text{jam}} - x_i), s_i^{\max}\}$$
$$f_i^{\text{in}}(x) = \sum_{j \in i^-} f_j^{\text{out}}(x)$$

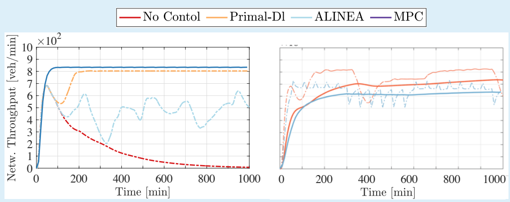
if traffic is controlled in free-flow

$$\dot{x} = Ax + Bu + Ew_t$$
$$y = Cx + Dw_t$$

Steady-state control, constrained to free-flow:

$$\min_{u,y} \quad (u - u^{\text{ref}})^\mathsf{T} Q_u (u - u^{\text{ref}}) - \Phi(y) \quad \text{(reference tracking)}$$
$$\text{s.t.} \quad y = -((R^\mathsf{T} - I)F)^{-1} Bu + w \quad \text{(steady-state map)}$$
$$u_i \geq 0, \quad y_i \leq \min\{x_i^{\text{crt},d}, x_i^{\text{crt},s}\} \quad \text{(free-flow traffic)}$$

Application to Los Angeles, CA, USA



without disturbances **with disturbances**

ALINEA:
Papageorgiou, Kotsialos, "Freeway ramp metering: An overview," *IEEE T-ITS*, 2002

Talk outline

1 – Making dynamic decisions
but the variability is measurable
and the variability is unmeasurable

2 - Making dynamic decisions in dynamic (!) environments

3 - Conclusions

Conclusions

- Increasingly fast dynamics in transportation
- Decision-making in dynamic environments ≠ classical optimization

Part 1:

- **Fundamental limitation 1**: tracking only if one has an internal model

Part 2:

- When transport system has non-negligible dynamics → "control loop"
- **Fundamental limitation 2**: optimization has to operate at a slower timescale than the plant

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fnrs LA LIBERTÉ DE CHERCHER | FÉDÉRATION WALLONIE-BRUXELLES | UCLouvain

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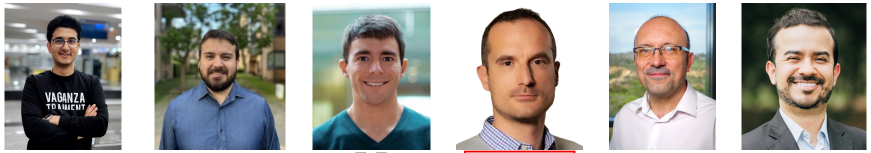

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Thank you!