Resilience of Traffic Networks with Partially Controlled Routing

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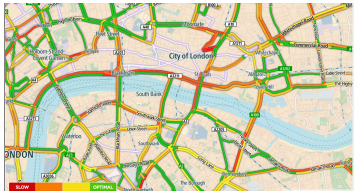
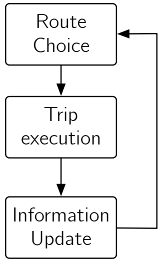
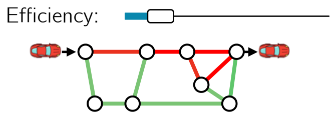
Routing in Traffic Networks

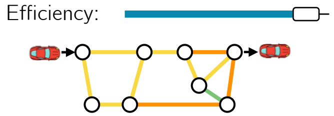


Network routing: captures how travelers respond to congestion





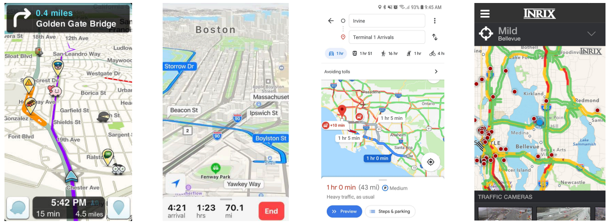
- Minimize individual delay
- Braess Paradox



- Do what's best for network
- Can do longer commute

The Introduction of Real-Time Traffic Information

- Commute as fast as we can



- Information does not necessarily make things better:


PCWorld

Researcher: Hackers can c
manipulating real-time traf

Hackers can influence real-time traffic
people drive into traffic jams or to kee
people use Google or Waze navigatio
demonstrated at BlackHat Europe.

Hackers Crack Into Tex
Warn of Zombies

Transportation officials in Texas are scrambling to prev
messages on digital road signs after one sign in Austir
Ahead."

Chris Lippincott, director of media relations for the Tex
confirmed that a portable traffic sign at Lamar Bouleva
University of Texas at Austin, was hacked into during tl

gt government technology

APPLICATIONS

Navigation App Waze Vulnerable to Hackers

Researchers have found that hackers can break into users accounts, track the users in real time, issue instructions and provide an inaccurate picture of traffic at any given time.

(TNS) -- A serious security breach has been discovered in the Google-owned navigation and traffic monitoring app Waze used by millions of drivers worldwide. It seems that Waze, whose slogan is "Outsmarting Traffic, Together" can be outsmarted by hackers, researchers at the University of California Santa Barbara have found, with serious privacy implications for users and scope for criminal abuse.

Two Emerging Questions

1. Do real-time routing suggestions improve performance?

✓ Yes, under appropriate design

2. What is the impact on the network robustness?

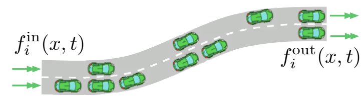
✗ Controlled routing can increase network fragility

Model of Network and Routing

Model of Traffic Network

- Roads dynamics

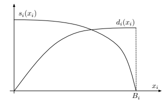
$$\dot{x}_i = f_i^{\text{in}}(x,t) - f_i^{\text{out}}(x,t)$$



x_i = density [veh/mile] f_i = flow [veh/h]

- Cell Transmission Model

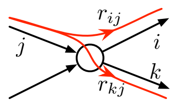
$$f_i^{\text{in}}(x,t) \le s_i(x_i) \quad f_i^{\text{out}}(x,t) \le d_i(x_i)$$



B_i = max occupancy

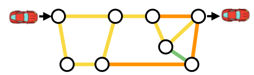
- Flow conservation at intersections

$$f_i^{\text{in}}(x,t) = \sum_j r_{ij} f_j^{\text{out}}(x,t)$$



- Network dynamics:

$$\dot{x} = (R - I)f(x,t) + \lambda$$

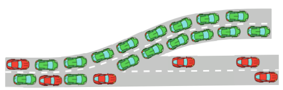


Partially Controlled Routing

$$R = \Sigma R^{\mathsf{C}} + (I - \Sigma)R^{\mathsf{S}}, \quad R^{\mathsf{S}} \in \mathcal{R}_{\mathcal{G}}, \quad R^{\mathsf{C}} \in \mathcal{R}_{\mathcal{G}}$$

- R^{C} = design parameter
- R^{S} = fixed routing
- $\Sigma = \text{Diag}\{\sigma_1, \dots, \sigma_n\}$

- $\sigma_i \in [0,1]$ describes penetration level at road i
- σ_i will follow suggestions r_{ij}^{c}
- $(1 - \sigma_i)$ will follow fixed routing r_{ij}^{s}



σ_i are link dependent and time-varying

Routing Design

1. Do real-time routing suggestions improve performance?

$$\begin{aligned}
\min_{R^{\mathsf{C}}} \quad & \int_0^{\mathcal{H}} x_1 + \dots + x_n \, dt \\
\text{s.t} \quad & \dot{x} = (R - I)f(x,t) + \lambda \\
& R = \Sigma R^{\mathsf{C}} + (I - \Sigma)R^{\mathsf{S}} \qquad (\sigma = \sigma_0) \\
& R^{\mathsf{C}} \in \mathcal{R}_{\mathcal{G}}, \quad x \le B
\end{aligned}$$

- Optimizing network total travel time
- Design parameter: R^{C} (routing suggestions)

Network Resilience

2. What is the impact real-time information on the network robustness?

$$\rho(\mathcal{G}, x_0) := \min_{\tilde{\sigma}} \quad \|\tilde{\sigma} - \sigma_0\|_1$$
$$\text{s.t.} \quad x_i = B_i, \ \text{for some } i \text{ and some } t$$

- Smallest change in penetration levels that results in spillbacks
- The variable parameter is: σ (penetration rate)

Routing Suggestions Design

Numerically Solving the Optimization

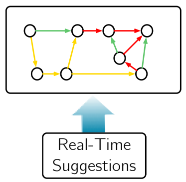
1. Discretization: $x_{k+1} = x_k + T_s((R_k - I)f(x_k) + \lambda_k) := \mathcal{F}(x_k, r_k, \lambda_k)$
2. Vectorization: $r_k = (\Sigma_k^{\mathsf{T}} \otimes I) r^{\mathsf{c}} + ((I - \Sigma_k)^{\mathsf{T}} \otimes I) r^{\mathsf{s}} := \Psi(\sigma_k, r^{\mathsf{s}}, r^{\mathsf{c}})$
3. Sparsity: $\sum_i r_{ij}^{\mathsf{c}} = b_j, \ 0 \leq r_{ij}^{\mathsf{c}} \leq 1, \quad (i,j) \in \mathcal{E}$

Optimization on discretized dynamics:

$$\begin{aligned}
\min_{r^{\mathsf{c}}} \quad & \sum_{k=1}^{h} \phi(x_k) \\
\text{s.t.} \quad & x_{k+1} = \mathcal{F}(x_k, r_k, \lambda_k), && k = 1, \dots, h, \\
& r_k = \Psi(\sigma_k, r^{\mathsf{s}}, r^{\mathsf{c}}), && k = 1, \dots, h, \\
& \sum_i r_{ij} = b_j, && j = 1, \dots, n, \\
& 0 \leq r_{ij}^{\mathsf{c}} \leq 1, && (i,j) \in \mathcal{E}, \\
& x_k \leq B, && k = 1, \dots, h,
\end{aligned}$$

Because of nonlinearity of $\mathcal{F}$: **nonlinear programming opt. problem**

Numerically Solving the Optimization



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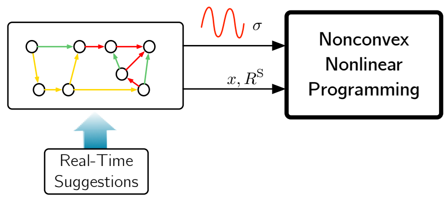


Optimization on discretized dynamics:

$$\min_{r^{\mathsf{c}}} \quad \sum_{k=1}^{h} \phi(x_k)$$

$$\begin{aligned}
\text{s.t.} \quad & x_{k+1} = \mathcal{F}(x_k, r_k, \lambda_k), && k = 1, \dots, h, \\
& r_k = \Psi(\sigma_k, r^{\mathsf{s}}, r^{\mathsf{c}}), && k = 1, \dots, h, \\
& \sum_i r_{ij} = b_j, && j = 1, \dots, n, \\
& 0 \leq r_{ij}^{\mathsf{c}} \leq 1, && (i,j) \in \mathcal{E}, \\
& x_k \leq B, && k = 1, \dots, h,
\end{aligned}$$

Because of nonlinearity of $\mathcal{F}$: **nonlinear programming opt. problem**

Solving Nonlinear Programming in Real Time

$$\begin{aligned}
\min_{r^{\mathsf{c}}} \quad & f_0(r^{\mathsf{c}}, \hat{x}, \sigma) \\
\text{s.t.} \quad & g_i(r^{\mathsf{c}}, \hat{x}, \sigma) \leq 0 \\
& h_j(r^{\mathsf{c}}, \hat{x}, \sigma) = 0
\end{aligned}$$

1. Solve NLP problem offline with $\sigma = \sigma_0$
2. Compose the Lagrangian

$$\mathcal{L}(r^{\mathsf{c}}, \hat{x}, \sigma, w, u) = f_0(r^{\mathsf{c}}, \hat{x}, \sigma) + u^{\mathsf{T}} g(r^{\mathsf{c}}, \hat{x}, \sigma) + w^{\mathsf{T}} h(r^{\mathsf{c}}, \hat{x}, \sigma)$$

3. Set of KKT conditions

Implicit Equation
$F(r^{\mathsf{c}*}, \hat{x}^*, \sigma_0, u^*, w^*) = 0,$

Describes Relation Between Variables At Optimality

$$\begin{aligned}
\nabla \mathcal{L}(r^{\mathsf{c}*}, \hat{x}^*, \sigma_0, w^*, u^*) &= 0 \\
u_i g_i(r^{\mathsf{c}*}, \hat{x}^*, \sigma_0) &= 0 \\
h_j(r^{\mathsf{c}*}, \hat{x}^*, \sigma_0) &= 0
\end{aligned}$$

Solving Nonlinear Programming in Real Time (2)

$F(r^{\mathsf{c}*}, \hat{x}^*, \sigma_0, u^*, w^*) = 0$

4. Let $y = [r^{\mathsf{c}}(\sigma) \ u(\sigma) \ w(\sigma)]$ and derive w.r.t. σ

$$M(\sigma) \frac{dy}{d\sigma} + N(\sigma) = 0,$$

$M(\sigma) = [\partial F_i / \partial y_j]$. $dy/d\sigma = [dy_i / d\sigma_j]$. $N(\sigma) = [\partial F_i / \partial \sigma_j]$

4. If $r^{\mathsf{c}*}(\sigma_0)$ is a local isolated minimizing point

$$\frac{dy}{d\sigma} = -M^{-1}(\sigma_0) N(\sigma_0)$$

Describes variation of optimal solution to changes in σ

5. Let $dy/d\sigma = [\eta_1, \ \eta_2, \ \eta_3]^{\mathsf{T}}$

$$r^{\mathsf{c}}(\sigma) = r^{\mathsf{c}*}(\sigma_0) + \eta_1(\sigma - \sigma_0)$$

Linear update: significantly faster than solving nonlinear programming

Fast Update Error Bound

Theorem

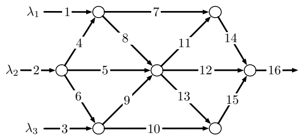
Assume

1. All constraints are linearly independent
2. Second order KKT conditions hold at optimizer
3. Strict complementary slackness

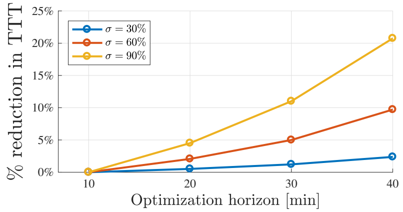
Then,

$$r^{\mathsf{c}*}(\sigma) = r^{\mathsf{c}*}(\sigma_0) + \eta_1(\sigma - \sigma_0) + o(\|\sigma - \sigma_0\|^2)$$

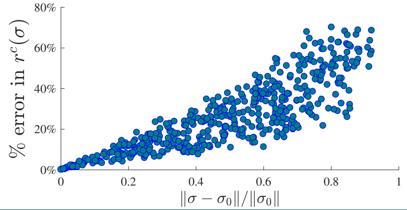
Numerical Results



- Network: $n = 15$ roads, $m = 7$ junctions
- Roads: $L_i = 5$mi, capacity $B_i = 200$veh
- Routing: r^{S}_{ij} split uniformly



Consistent reduction in travel time not only requires significant penetration levels, but also appropriately large control horizons



Error with real-time update = $o(\|\sigma - \sigma_0\|^2)$

Network Resilience

Resilience of a Link

The Margin of Resilience of link i is

Smallest change in σ that generates its failure due to a jam

$$\begin{aligned} \rho_i(x_0) := \min_{\sigma} \quad & \|\sigma - \sigma_0\|_1 \\ \text{s.t.} \quad & \dot{x} = (R - I) f(x,t) + \lambda \\ & R = \Sigma R^{\mathsf{C}} + (I - \Sigma) R^{\mathsf{S}} \\ & x_i \geq B_i, \ \text{for some } t \in [0, \mathcal{H}] \end{aligned}$$

Estimating the Margin of Resilience

1. Constraint violation is $\mathcal{F}_i(x_k, r_k(\sigma), \lambda_k) \geq B_i$
2. For small changes of σ, take Taylor expansion

$$\mathcal{F}_i(x_k, r_k(\sigma), \lambda_k) = \mathcal{F}_i(x_k, r_k(\sigma_0), \lambda_k) + \underbrace{\frac{d\mathcal{F}_i}{d\sigma}(x_k, r_k(\sigma), \lambda_k)\Big|_{\sigma=\sigma_0}}_{\Psi_i(r_k, x_k, \lambda_k, \sigma)} (\sigma - \sigma_0) + o(\|\delta_\sigma\|^2) \geq B_i$$

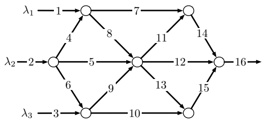
3. By rearranging the terms and by taking the norms

$$\rho_i(\mathcal{G}, x_0) \geq \min_k \frac{B_i - \mathcal{F}_i(x_k, r_k, \lambda_k)}{\|\Psi_i(k, \lambda, \sigma_0)\|_\infty}$$

Lower Bound on Margin of Resilience

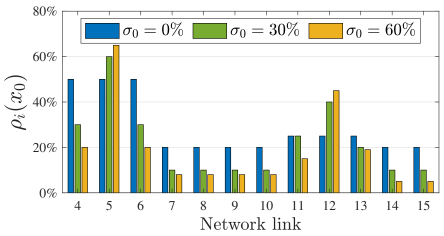
(Can be quickly computed from real-time update)

Numerical Results



- Network: $n = 15$ roads, $m = 7$ junctions
- Roads: $L_i = 5$mi, capacity $B_i = 200$veh
- Routing: r_{ij}^{s} split uniformly

Lower bound on resilience:



Networks where the routing is partially controlled by a system planner are more prone to traffic jam phenomena originated by changes in fluctuations in routing choices

Conclusions

Contribution:

- Real-time mechanism based on first-order sensitivity analysis for NLP
- Technique to estimate margin of resilience

Outcomes:

- (Expected) Routing control leads to improved network performance
- (Counterintuitive) Performance comes at the cost of higher fragility

Directions:

✓ Need of a framework that allows us to formalize these observations

✎ Increasing need to capture human responses to control policies

Thanks: Control of Complex Systems Initiative (CCSI) at PNNL & NSF

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