A Network Optimization Approach for the Optimization of Intersection Signaling

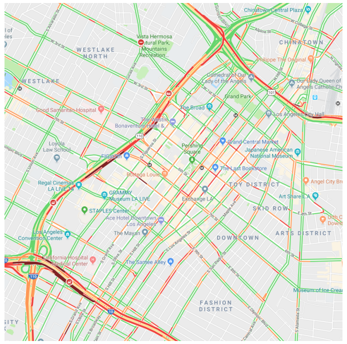
Gianluca Bianchin, Fabio Pasqualetti

Department of Mechanical Engineering
University of California, Riverside

May 11, 2018 | SoCal Control Workshop | UC Riverside

Motivation

- Transportation: critical infrastructure for development of smart cities
- High complexity
- Efficiency strongly depend on **control of traffic signaling**
- Current control techniques rely on infrastructure sensing
- Intelligent vehicle technologies:
  - **New layer of communication**
  - Enormous potential for control

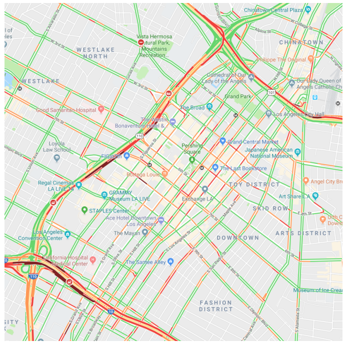


Google live traffic, Downtown LA

Goal

minimize (lights schedule)	(network congestion)
subject to	(traffic conditions)
	(network interconnection)

- Current methods: distributed
  - Local (infrastructure) sensing
  - Scale well
- Centralized
  - Use V2I and I2V communication
  - Insights for new control variables
  - Higher complexity

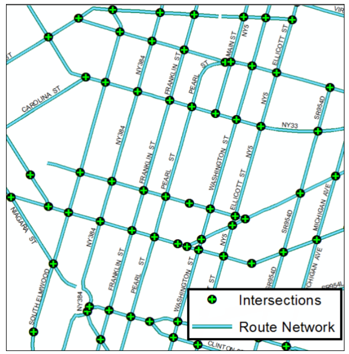


Google live traffic, Downtown LA

Trade-off between model complexity and performance

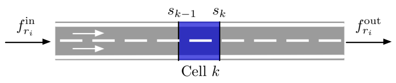
Model of traffic network

- Traffic network described by $\mathcal{N} = (\mathcal{R}, \mathcal{I})$
  - $\mathcal{R} = \{r_1, \dots, r_{n_r}\}$ set of one-way roads
  - $\mathcal{I} = \{\mathcal{I}_1, \dots, \mathcal{I}_{n_\mathcal{I}}\}$ intersections
- Exogenous inflows and outflows
  - Enter at source roads $\mathcal{S} \subseteq \mathcal{R}$
  - Exit at destination roads $\mathcal{D} \subseteq \mathcal{R}$



Modeling roads

$$\frac{\partial \rho}{\partial t} + \frac{\partial f}{\partial s} = 0$$



Cell Transmission Model

$$\dot{x}^k = \frac{\gamma}{h}(x^{k-1} - x^k)$$

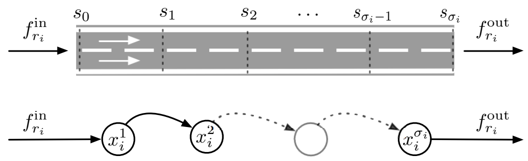
- x^k density of cell k
- γ average speed of the flow
- h discretization step

- Road dynamics:

$$\begin{bmatrix} \dot{x}_i^1 \\ \dot{x}_i^2 \\ \vdots \\ \dot{x}_i^{\sigma_i} \end{bmatrix} = \frac{\gamma_i}{h} \begin{bmatrix} -1 & & & \\ 1 & -1 & & \\ & \ddots & \ddots & \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x_i^1 \\ x_i^2 \\ \vdots \\ x_i^{\sigma_i} \end{bmatrix} + \begin{bmatrix} f_{r_i}^{\text{in}} \\ 0 \\ \vdots \\ -f_{r_i}^{\text{out}} \end{bmatrix}$$

Modeling roads

Analogy between road and the associated network model



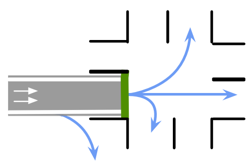
- Road Dynamics

$$\begin{bmatrix} \dot{x}_i^1 \\ \dot{x}_i^2 \\ \vdots \\ \dot{x}_i^{\sigma_i} \end{bmatrix} = \frac{\gamma_i}{h} \begin{bmatrix} -1 & & & \\ 1 & -1 & & \\ & \ddots & \ddots & \\ & & 1 & 0 \end{bmatrix} \begin{bmatrix} x_i^1 \\ x_i^2 \\ \vdots \\ x_i^{\sigma_i} \end{bmatrix} + \begin{bmatrix} f_{r_i}^{\text{in}} \\ 0 \\ \vdots \\ -f_{r_i}^{\text{out}} \end{bmatrix}$$

Model of interconnection flows

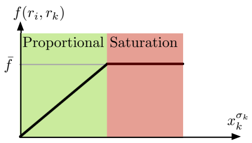
Intersections control road outflows

$$f_{r_i}^{\text{out}} = \sum_{r_k} s(r_k, r_i, t)\, f(r_k, r_i)$$



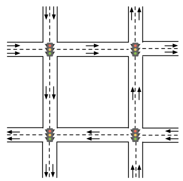
$$f(r_i, r_k) = \min\{c(r_i, r_k) x_k^{\sigma_k}, \bar{f}\}$$

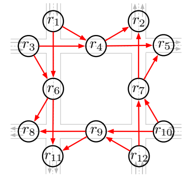
- Green split $s : \mathcal{R} \times \mathcal{R} \times \mathbb{R}_{\geq 0} \to \{0, 1\}$
- Transmission rate $f : \mathcal{R} \times \mathcal{R} \to \mathbb{R}_{\geq 0}$



$$f(r_i, r_k) \approx c(r_i, r_k) x_k^{\sigma_k}$$

Network model





$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n_r} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1n_r} \\ A_{21} & A_{22} & \ddots & A_{2n_r} \\ \vdots & \ddots & \ddots & \vdots \\ A_{n_r 1} & A_{n_r 2} & \cdots & A_{n_r n_r} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n_r} \end{bmatrix}$$

- Outflows: proportional to roads occupancy

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{n_r} \end{bmatrix} = \begin{bmatrix} e_{\sigma_1}^{\mathsf{T}} & \dots & 0 \\ \vdots & \ddots & \\ 0 & \dots & e_{\sigma_{n_r}}^{\mathsf{T}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n_r} \end{bmatrix}$$

Queue lengths are cell densities at the downstream

A linear switching system

Linear switching system, where the switching signals are the green split functions $s(r_i, r_k, t)$

$$\dot{x} = A_{s(r_i,r_k,t)}x$$
$$y = Cx$$

Problem formulation

Design problem

- Assume the network has a certain initial density x_0
- Find the green split functions that minimize the queue lengths

$$\dot{x} = A_{s(r_i,r_k,t)}x$$
$$y = Cx$$

Problem formulation

Design problem

- Assume the network has a certain initial density x_0
- Find the green split functions that minimize the queue lengths

$$\min_{s(r_i,r_k,t)} \int \|y\|_2^2 \, dt$$

s.t.
$$\dot{x} = A_{s(r_i,r_k,t)}x$$
$$y = Cx$$
$$x(0) = x_0$$
$s(r_i, r_k, t)$ is a feasible green split

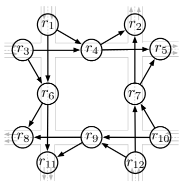
Approximating traffic switching system

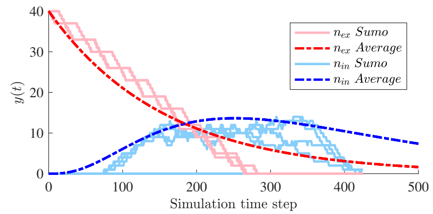
$$\dot{x} = A_{s(r_i,r_k,t)}x$$
$$s(r_i, r_k, t) = \text{piecewise constant}$$

Define $\{d_1, \dots, d_m\}$ durations where $s(r_i, r_k, t) = $ constant

$$\dot{x}_{\mathsf{av}} = A_{\mathsf{av}}x_{\mathsf{av}}$$
$$A_{\mathsf{av}} = \frac{1}{T}(A_1 d_1 + \cdots + A_m d_m)$$

"Average" network dynamics





Mode durations $\{d_1, \dots, d_m\}$ are the new design parameters

A network optimization problem

$$\min_{d_1,\dots,d_m} \int_0^\infty \|y_{\text{av}}\|_2^2 \, dt$$

subject to
$$\dot{x}_{\text{av}} = A_{\text{av}} x_{\text{av}}$$
$$y_{\text{av}} = C_{\text{av}} x_{\text{av}}$$
$$A_{\text{av}} = \frac{1}{T}(d_1 A_1 + \cdots + d_m A_m)$$
$$x_{\text{av}}(0) = x_0$$
$$T = d_1 + \cdots + d_m$$
$$d_i \geq 0 \quad i \in \{1, \dots, m\}$$

- Measurements will enter the optimization, updating x_0
- We optimize over $[0, \infty]$ and follow a "receding horizon" approach

Cost function

$$\min_{d_1,\dots,d_m} \int_0^\infty \|y_{\text{av}}\|_2^2 \, dt = \min_{d_1,\dots,d_m} \int_0^\infty x_0^\mathsf{T} e^{A_{\text{av}}^\mathsf{T} t} C_{\text{av}}^\mathsf{T} C_{\text{av}} e^{A_{\text{av}} t} x_0 \, dt$$

- Finite if exist $\{d_1, \dots, d_m\}$ that lead to A_{av} Hurwitz

(Theorem) Network stability = Graph-theoretic property

If there exists a path in $\mathcal{N}$ between any source $s \in \mathcal{S}$ and some destination $d \in \mathcal{D}$, then there exists $\{d_1, \dots, d_m\}$:

$$\alpha(A_{\text{av}}) < 0$$

Spectral abscissa of A_{av}

$$\alpha(A_{\text{av}}) := \sup\{\Re(s) : s \in \mathbb{C}, \det(sI - A_{\text{av}}) = 0\}$$

Controllability metrics of traffic networks

- Define the Gramian matrix

$$\mathcal{W}(A_{\text{av}}, x_0) = \int_0^\infty e^{A_{\text{av}} t} x_0 x_0^\mathsf{T} e^{A_{\text{av}}^\mathsf{T} t} \, dt$$

(Lemma) Network performance and controllability

$$\min_{d_1,\dots,d_m} \operatorname{Trace}\left(C_{\text{av}} \, \mathcal{W}(A_{\text{av}}, x_0) \, C_{\text{av}}^\mathsf{T}\right)$$

subject to
$$A_{\text{av}} = \frac{1}{T}(d_1 A_1 + \cdots + d_m A_m)$$
$$T = d_1 + \cdots + d_m$$
$$d_i \geq 0, \quad i \in \{1, \dots, m\}$$

The optimal split durations minimize
a controllability metric

Optimizing network controllability

$$\min_{d_1,\dots,d_m} \operatorname{Trace}\left(C_{\text{av}} \, \mathcal{W}(A_{\text{av}}, x_0) \, C_{\text{av}}^\mathsf{T}\right)$$

Difficulties:

- A_{av} and $\mathcal{W}(A_{\text{av}}, x_0)$ are related by the (**nonlinear**) relation

$$A_{\text{av}} \, \mathcal{W} + \mathcal{W} \, A_{\text{av}}^\mathsf{T} = -(x_0 x_0^\mathsf{T})(x_0 x_0^\mathsf{T})$$

- Similar problems: consider stability $\alpha(A_{\text{av}})$
  - Captures steady state rates (not transient overshoots)
  - Nonconvex in A_{av} and "very hard to optimize"

J. Vanbiervliet, B. Vandereycken, W. Michiels, S. Vandewalle, and M. Diehl, "The smoothed spectral abscissa for robust stability optimization," in *SIAM Journal on Optimization*, vol. 20, no. 1, 2009.

Optimizing network controllability

- For a certain A_{av} the associated network performance is

$$\mathrm{Trace}\left(C_{\text{av}}\ \mathcal{W}(A_{\text{av}}, x_0)\ C_{\text{av}}^{\mathsf{T}}\right) = 1/\epsilon$$

- Now assume we desire a better performance $\bar{\epsilon} > \epsilon$
- We can make the system "faster" $A_{\text{av}} \to A_{\text{av}} - sI$, $s \in \mathbb{R}$ variable

$$\mathrm{Trace}\left(C_{\text{av}}\ \mathcal{W}(A_{\text{av}} - sI, x_0)\ C_{\text{av}}^{\mathsf{T}}\right) = 1/\bar{\epsilon}$$

- Solution $s := \alpha_\epsilon(A_{\text{av}})$: "smoothed spectral abscissa"
  - Unique
  - Differentiable in A_{av} (and $\{d_1, \dots d_m\}$)

Questions

- Can we design A_{av} so that $s = \alpha_{\bar{\epsilon}}(A_{\text{av}}) = 0$?
- If yes, A_{av} will have performance $1/\bar{\epsilon}$

Optimizing network controllability

$$\begin{aligned} \alpha_{\bar{\epsilon}}^* = \min_{d_1,\dots,d_m} \quad & |\alpha_{\bar{\epsilon}}(A_{\text{av}})| \\ \text{subject to} \quad & A_{\text{av}} = \frac{1}{T}(d_1 A_1 + \dots + d_m A_m) \\ & T = d_1 + \dots + d_m \\ & d_i \geq 0, \quad i \in \{1, \dots, m\} \end{aligned}$$

Gradient descent can numerically solve this problem

$$\frac{\partial \alpha_\epsilon(A_{\text{av}})}{\partial d} = \mathrm{vec}\left(\frac{QP}{\mathrm{Trace}\,(QP)}\right)\frac{\partial a_{\text{av}}}{\partial d}$$

$$(A_{\text{av}} - \alpha_\epsilon(A_{\text{av}})I)P + P(A_{\text{av}} - \alpha_\epsilon(A_{\text{av}})I)^{\mathsf{T}} + x_0 x_0^{\mathsf{T}} = 0$$

$$(A_{\text{av}} - \alpha_\epsilon(A_{\text{av}})I)^{\mathsf{T}} + Q(A_{\text{av}} - \alpha_\epsilon(A_{\text{av}})I) + C_{\text{av}} C_{\text{av}}^{\mathsf{T}} = 0$$

Optimizing network controllability: gradient-descent algorithm

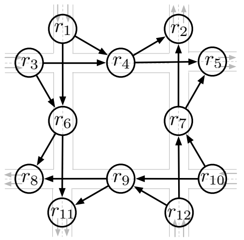
$$\begin{aligned} \min_{d_1,\dots,d_m} \quad & |\alpha_{\bar{\epsilon}}(A_{\text{av}})| \\ \text{s.t.} \quad & A_{\text{av}} = \frac{1}{T}(d_1 A_1 + \dots + d_m A_m) \\ & T = d_1 + \dots + d_m \\ & d_i \geq 0, \quad i \in \{1, \dots, m\} \end{aligned}$$

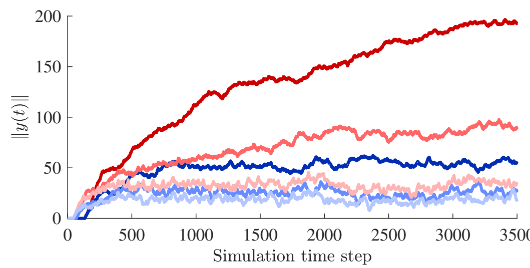
Input: Matrix C_{av}, vector x_0, scalars μ, ν_{min}, $\bar{\epsilon}$
Output: $\{d_1^*, \dots d_m^*\}$
Initialize $d^{(0)}$, $k = 1$
repeat
  Compute the current value of $\alpha_\epsilon(A_{\text{av}}^{(k)})$;
  Compute $\frac{\partial \alpha_\epsilon(A_{\text{av}}^{(k)})}{\partial d}$;
  Compute $\frac{\partial \alpha_\epsilon(A_{\text{av}}^{(k)})^2}{\partial d^{(k)}}$;
  Update: $\delta^{(k)} \leftarrow d^{(k-1)} + \mu\,\frac{\partial \alpha_\epsilon(A_{\text{av}}^{(k)})^2}{\partial d^{(k)}}$;
  Projection: $d^{(k)} \leftarrow \underset{d \in \Delta}{\mathrm{argmin}} \|\delta^{(k)} - d\|$;
  Update $A_{\text{av}}^{(k)}$;
  $k \leftarrow k + 1$;
until $|\alpha_\epsilon(A_{\text{av}}^{(k)})^2 - \alpha_\epsilon(A_{\text{av}}^{(k-1)})^2| < \nu_{\text{min}}$;
return $d^{(k)}$;

Preliminary simulation experiments

Additional algorithm complexity is justified by increased performance

Distributed vs Centralized





Dark colors: "slower" network
Light colors: "faster" network

Summary and ongoing effort

Motivation: incorporate new traffic data and model interconnection

Approximate model: tradeoff between complexity and accuracy

- Benefits: centralized techniques give better insight
- Allow network design
- Allow design of new control parameters
- Better performance

Ongoing effort:

- Validation of the technique over existing traffic networks
- Computational:
  - Distributed implementation of gradient descent
- Security analysis