Online Feedback Optimization with Applications to Transportation Networks

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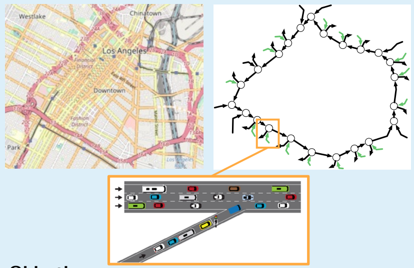
Motivation: control and coordination of transportation

Ramp metering



Objective:

Control ramp flows to maximize throughput

Difficulties:

- Complex, nonlinear, network model
- Rapid changes in traffic demand at peak hours

How is currently done:

- PI-type controller, local at ramps/corridors
- Simulation-based controller tuning, seasonal parameters update

Ride-hailing scheduling

Objective:

Decide ride prices to maximize company's profit

Difficulties:

- Humans–in-the-loop: demand elasticity vs prices
- Highly dynamic ride requests & vehicle availability

How is currently done:

- Customer-vehicle bipartite matching, locally

Problem setting

Online Feedback Optimization

Design a controller that optimizes the operation of the system in the face of unknown disturbances

Physical system:

$$x_{k+1} = Ax_k + Bu_k + Ew_k$$
$$y_k = Cx_k + Dw_k$$

Noise model:

$w_k \sim \mathcal{W}_k$ (time-varying distribution)

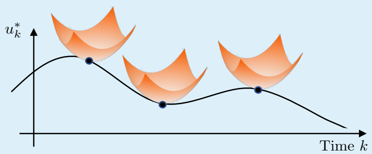
Steady-state map:

$$y = \underbrace{C(I-A)^{-1}B}_{G}\,u + \underbrace{(D + C(I-A)^{-1}E)}_{H}\,w$$

Objective: (equilibrium selection)

$$\min_{u^d} \; \mathbb{E}_{w_k}[\phi(u^d, \underbrace{Gu^d + Hw_k}_{\text{steady-state output}})]$$

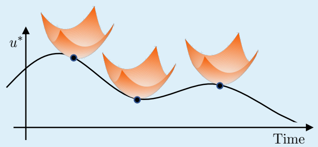
- Implicitly defines optimal operating point for the system affected by current disturbance
- Challenge: noise is unmeasurable thus the optimization cannot be solved explicitly



Control objective: related work

Control objective:

$$\min_{u^d} \; \mathbb{E}_{w_k}[\phi(u^d, Gu^d + Hw_k)]$$

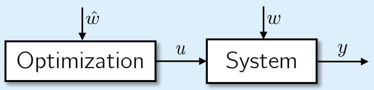


Conceptually-related problems:

- **Output regulation:** control system so that output tracks a prescribed reference
  [Davidson '76], [Francis '77], [Yoon and Lin 16], ..., [Huang '03,'04], [Isidori '89], ...
- **Extremum-seeking:** seek the extremum of a performance metric, adjusting control inputs online
  [Leblanc, '22], ... [Wittenmark & Urquhart, '95], ... [Krstić & Wang, '00], ... , [Feiling et.al., '18]
- **MPC (real-time/online):** more general control objective, but harder to solve
  Real-time MPC [Zeilinger et.al. '09], Optimizing control [Garcia & Morari '81], ...
- **Optimal control (e.g., LQR)**: more general control objective, requires noise knowledge
  [Bertsekas '95], ...

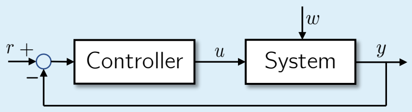
Motivation

Batch optimization



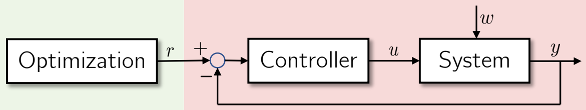
- **Complex specifications**
  optimality, constraints, accounts transient, ...
- **High sensitivity to disturbances**
  (accuracy depends on predicted disturbance)
- **Computationally challenging**
  must solve an optimization problem

Feedback control



- Simple specifications
- Disturbance rejection
- Controller computed once offline

- The two approaches are often combined as follows:

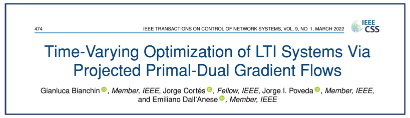


Slow timescale
(days, weeks, ...)

Fast timescale
(hours, minutes, seconds, milliseconds, ...)

Talk outline

1. Algorithm design and tracking analysis
2. Direct design from data
3. Continuous-time and constrained optimization


474 IEEE TRANSACTIONS ON CONTROL OF NETWORK SYSTEMS, VOL. 9, NO. 1, MARCH 2022

Time-Varying Optimization of LTI Systems Via Projected Primal-Dual Gradient Flows

Gianluca Bianchin, Member, IEEE, Jorge Cortés, Fellow, IEEE, Jorge I. Poveda, Member, IEEE, and Emiliano Dall'Anese, Member, IEEE

G. Bianchin, J. Cortés, J. I. Poveda and E. Dall'Anese, "Time-Varying Optimization of LTI Systems via Projected Primal-Dual Gradient Flows," IEEE TCNS, 2022


Online Stochastic Optimization for Unknown Linear Systems: Data-Driven Controller Synthesis and Analysis

Gianluca Bianchin, Miguel Vaquero, Jorge Cortés, and Emiliano Dall'Anese

G. Bianchin, M. Vaquero, J. Cortés and E. Dall'Anese, "Online Stochastic Optimization for Unknown Linear Systems: Data-Driven Synthesis and Controller Analysis," IEEE TAC, (to appear)

Algorithms and closed-loop error tracking analysis

Algorithm design idea

Review of classical optimization tools:

- Consider the **stochastic optimization problem**:

$$\min_{u^d} \; \mathbb{E}_{w_k}[\phi(u^d, Gu^d + Hw_k)]$$

- **Gradient descent** iteration:

$$\text{(compute:)} \;\; \Phi(u_k) = (\nabla_u \circ \mathbb{E}_{w_k} \circ \phi)(u_k, Gu_k + Hw_k)$$
$$u_{k+1} = u_k - \eta \Phi(u_k)$$

- When distribution is unknown, **stochastic gradient descent:**

$$\text{(sample:)} \;\; y_k = Gu_k + Hw_k$$
$$\text{(compute:)} \;\; \Phi^s(u_k, y_k) := (\nabla_u \circ \phi)(u_k, y_k)$$
$$u_{k+1} = u_k - \eta \Phi^s(u_k, y_k)$$

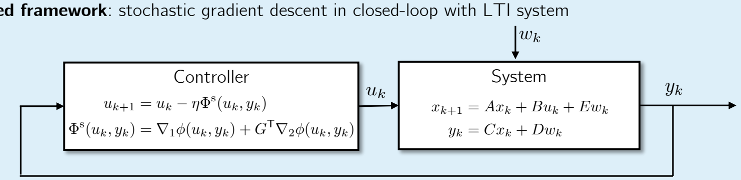
System to control:

$$x_{k+1} = Ax_k + Bu_k + Ew_k$$
$$y_k = Cx_k + Dw_k$$

with steady-state map:

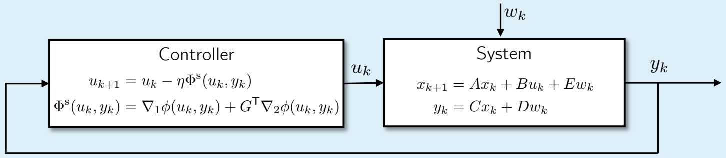
$$y = Gu + Hw$$

Proposed framework: stochastic gradient descent in closed-loop with LTI system



Online optimization as a feedback controller

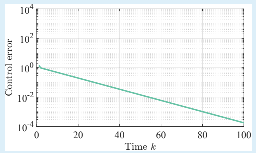
Proposed control framework:



What if we "feed" the state of a gradient-descent algorithm as input to a physical system?

Algebraic system model:

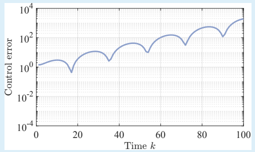
$u_k = u_{k-1} - \nabla \Phi(u_{k-1}, y_k) \qquad y_k = Gu_k + Hw_k$



Dynamic system model:

$u_k = u_{k-1} - \nabla \Phi(u_{k-1}, y_k) \qquad x_{k+1} = Ax_k + Bu_k + Ew_k$

$y_k = Cx_k + Dw_k$



Closed-loop tracking bound

Theorem: Assume

- Regularity of ϕ: ℓ-Lipschitz gradient and μ-strongly convex
- Plant properties: Controllability + Observability + Stability: $\exists P, Q: \; A^\mathsf{T} P A - P = -Q$
- Define:

$$e_k^* := \left\| \begin{bmatrix} u_k \\ x_k \end{bmatrix} - \begin{bmatrix} u_k^* \\ x_k^* \end{bmatrix} \right\| \qquad \text{samp-err}_k := (\nabla_u \circ \mathbb{E}_{w_k} \circ \phi)(u_k, Gu_k + Hw_k) - (\nabla_u \circ \phi)(u_k, Gu_k + Hw_k)$$

- Then, for any $k \geq 0$:

$$\mathbb{E}[e_{k+1}^*] \leq \underbrace{\beta \mathbb{E}\,[e_k^*]}_{\text{contraction}} + \underbrace{\gamma \mathbb{E}[\sup_{0 \leq \tau \leq k} \|e_{\tau+1}^* - e_\tau^*\|]}_{\text{temporal shift of optimizer}} + \underbrace{\eta \mathbb{E}[\|\text{samp-err}_k\|]}_{\text{sampling error}}$$

where $\beta = \max\{ \underbrace{\sqrt{1-\eta\mu}}_{\text{Impose} < 1}, \underbrace{\sqrt{\frac{\bar\lambda(P)}{\underline\lambda(P)}\left(1 - \frac{(1-\kappa)\underline\lambda(Q)}{\bar\lambda(P)}\right)} + \eta \ell^\nabla \|C\|}_{< 1 \text{ if open-loop plant is contractive}} \}$

Implications:

- If $\eta < \dfrac{1}{\mu}$, tracking error is contractive!
- Error bound depends on **temporal variability of the optimizer** and on **sampling error**

Closed-loop stability analysis

Proof sketch of:

$$\mathbb{E}[e_{k+1}^*] \leq \beta \mathbb{E}\,[e_k^*] + \gamma \mathbb{E}[\sup_{0 \leq \tau \leq k} \|e_{\tau+1}^* - e_\tau^*\|] + \eta \mathbb{E}[\|\text{samp-err}_k\|]$$

The proof combines **gradient-descent contraction bounds** with **Lyapunov stability** bounds for LTIs

$$\mathbb{E}[e_{k+1}^*] \leq \underbrace{\mathbb{E}[\|u_{k+1} - u_{k+1}^*\|]}_{\text{contraction of stochastic gradient-descent}} + \underbrace{\mathbb{E}[\|x_{k+1} - x_{k+1}^*\|]}_{\text{contraction of system state (Lyapunov-like analysis)}}$$

Test case: ride-service scheduling in Manhattan

Ride-hailing model

- Discretize region into pickup/dropoff areas
- Mass-conservation law:

$$x_{k+1}^i = \underbrace{x_k^i - \sum_{j \in \mathcal{V}} a_{ij} x_k^i + \sum_{j \in \mathcal{V}} a_{ji} x_k^j}_{\text{conservation}} - \underbrace{\sum_{j \in \mathcal{V}} d_k^{ij}}_{\text{pick ups}} + \underbrace{\sum_{j \in \mathcal{V}} \sum_{\tau = k-T}^{k-1} \sigma_\tau^{ji,k-\tau} d_\tau^{ji} + e_k^i}_{\text{drop offs}}$$

x_k^i = idle vehicles in region i at time k

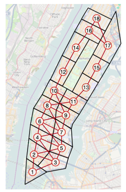
Optimization objectives

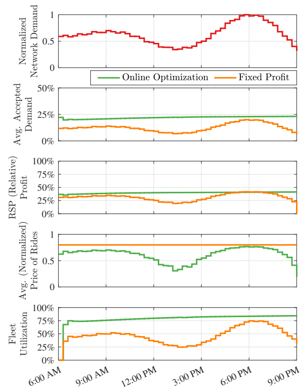
Ride demand is elastic to prices: $d_k^{ij} = \delta_k^{ij} \left(1 - \theta^{ij} \dfrac{p_k^{ij}}{p_{\max}^{ij}}\right)$

Control goal: set ride prices to maximize profit:

$$\begin{aligned}
\max_{p,x,d} \quad & \sum_{i \in \mathcal{V}} \sum_{j \in \mathcal{V}} p^{ij} d^{ij} - c^{ij} d^{ij} - \varrho \|x\|^2, && \text{(maximize profit)} \\
\text{s.t.} \quad & 0 = -\sum_{j \in \mathcal{V}} a_{ij} x^i + \sum_{j \in \mathcal{V}} a_{ji} x^j - \sum_{j \in \mathcal{V}} d^{ij} + w_k^i, && \text{(steady-state map)} \\
& d^{ij} = \delta_k^{ij} \left(1 - \theta^{ij} p^{ij} / p_{\max}^{ij}\right) && \text{(demand elasticity)} \\
& d^{ij} \geq 0, \; x^i \geq 0 && \text{(feasibility)}
\end{aligned}$$

Test scenario from Manhattan, NY



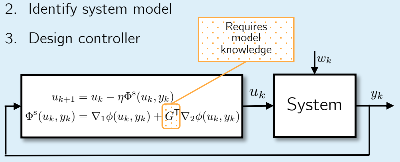


Data-based impementation

Algorithm design idea

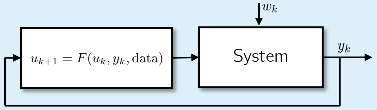
Model-based feedback optimization

1. Collect data
2. Identify system model
3. Design controller



Data-based feedback optimization

1. Collect data
2. Design controller



Necessary notation

- Assume availability of historical data: $y_{[0,T]},\ u_{[0,T]}$
- Compute finite differences: $y^{\text{diff}} = (y_1 - y_0,\ y_2 - y_1, \dots,\ y_T - y_{T-1})$
- For a signal $z_{[0,T-1]}$, define Hankel matrix: $Z_{t,q} = \begin{bmatrix} z_0 & z_1 & \dots & z_{q-1} \\ z_1 & z_2 & \dots & z_q \\ \vdots & \vdots & \ddots & \vdots \\ z_{t-1} & z_t & \dots & z_{q+t-2} \end{bmatrix}$

Proposed data-based implementation:

direct computation of G^T from data

$$u_{k+1} = u_k - \eta\Big(\nabla_1\phi(u_k, y_k) + \boxed{\Big([Y_{\nu,q}]_i \begin{bmatrix} Y^{\text{diff}}_{\nu,q} \\ U_{\nu,q} \end{bmatrix}^\dagger \begin{bmatrix} 0 \\ \mathbb{1}_\nu \otimes I_m \end{bmatrix}\Big)^\mathsf{T}}\ \nabla_2\phi(u_k, y_k)\Big)$$

Theoretical guarantees

Theorem. Assume

- System is controllable
- (joint) Input is persistently exciting of order $n + \nu$, $n =$state size, $\nu =$ observability index
- If **historical data is noiseless:**

$$C(I - A)^{-1}B = [Y_{\nu,q}]_i \begin{bmatrix} Y^{\text{diff}}_{\nu,q} \\ U_{\nu,q} \end{bmatrix}^\dagger \begin{bmatrix} 0 \\ \mathbb{1}_\nu \otimes I_m \end{bmatrix} := \hat{G}$$

- If **historical data is noisy:**

$$G - \hat{G} = [Y_{\nu,q}]_i \left(\begin{bmatrix} Y^{\text{diff}}_{\nu,q} \\ U_{\nu,q} \\ W_{\nu,q} \\ W^{\text{diff}}_{\nu,q} \end{bmatrix}^\dagger \begin{bmatrix} 0 \\ \mathbb{1}_\nu \otimes I_m \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} Y^{\text{diff}}_{\nu,q} \\ U_{\nu,q} \end{bmatrix}^\dagger \begin{bmatrix} 0 \\ \mathbb{1}_\nu \otimes I_m \end{bmatrix} \right)$$

Proof sketch. Assume $C = I$, so that $\nu = 1$

- By the fundamental lemma: $\operatorname{rank} \begin{bmatrix} U_{\nu,q} \\ X_{1,q} \end{bmatrix} = \nu m + n$ and $\begin{bmatrix} \tilde{u}_{[0,\nu-1]} \\ \tilde{y}_{[0,\nu-1]} \end{bmatrix} = \begin{bmatrix} U_{\nu,q} \\ Y_{\nu,q} \end{bmatrix} \alpha$
- At equilibrium: $0 = [I - A \quad -B] \begin{bmatrix} y^{\text{eq}} \\ u^{\text{eq}} \end{bmatrix} = [I - A \quad -B] \begin{bmatrix} Y_{\nu,q} \\ U_{\nu,q} \end{bmatrix} \alpha = Y^{\text{diff}}_{\nu,q}\alpha$

Thus, an equilibrium trajectory is characterized by the manifold $0 = Y^{\text{diff}}_{\nu,q}\alpha$

- Moreover, among all equilibrium trajectories, we want: $U_{\nu,q}\alpha = I$

Fundamental lemma:
Willems, Rapisarda, Markovsky, De Moor, "A note on persistency of excitation," Systems & Control Letters, 2005

Direct vs indirect estimation of G

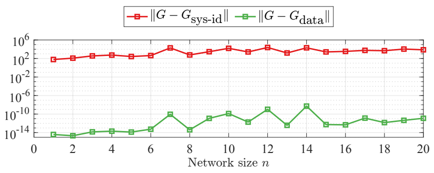
Direct method

$$\hat{G} = [Y_{\nu,q}]_i \begin{bmatrix} Y^{\text{diff}}_{\nu,q} \\ U_{\nu,q} \end{bmatrix}^\dagger \begin{bmatrix} 0 \\ \mathbb{1}_\nu \otimes I_m \end{bmatrix}$$

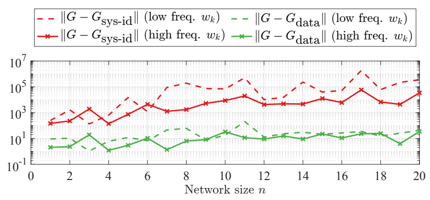
Indirect method (subspace id – [Ljung '99])

1. Estimate observability matrix $\hat{\mathcal{O}}$
2. From $\hat{\mathcal{O}}$, estimate $\hat{A}$ and $\hat{C}$
3. From $\hat{\mathcal{O}}$, estimate $\hat{x}(t)$
4. From $\hat{x}(t)$ and $\hat{\mathcal{O}}$, estimate $\hat{B}$ and $\hat{D}$, solving a linear regression
5. Compute: $\hat{G} = \hat{C}(I - \hat{A})^{-1}\hat{B}$

Noiseless data



Noisy data



Lesson learned from the numerics:

- Subspace id needs several assumptions: known system order, minimal realization
- Subspace id suffers from: many choices state variables, noise "propagates" through sequential id steps

The cost of an imprecise model description

If G is known with **exact precision**:

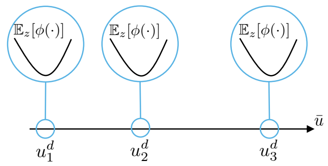
$$\min_{u^d} \quad \mathbb{E}_w[\phi(u^d, Gu^d + Hw)]$$

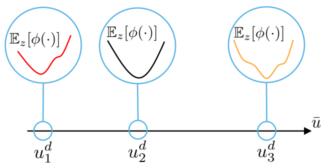
is a **classical** stochastic optimization problem

If G is known **approximately** as $\hat{G}$:

$$\min_{u^d} \quad \mathbb{E}_z[\phi(u^d, \hat{G}u^d + \underbrace{(G-\hat{G})u^d + Hw}_{z(u^d)})]$$

optimization **decision-dependent distribution**





Notion of stable optimizer:

A **stable optimizer** is a fixed point of the map:

$$u^{\text{so}} = \arg\min_{u^d} \quad \mathbb{E}_z[\phi(u^d, \hat{G}u^d + z(u^{\text{so}})]$$

it is a point that solves the optimization, provided that the distribution remains fixed

J. C. Perdomo, T. Zrnic, C. Mendler-Dunner, and M. Hardt, "Performative prediction," International Conference on Machine Learning, pp. 7599-7609, 2020

Lemma: Assume that

- $y \mapsto \phi(u, y)$ is ℓ-Lipschitz
- $u \mapsto \phi(u, y)$ is μ-strongly convex
- Let $\sigma^2_{\min}(\hat{G})$ be the smallest singular value of $\hat{G}$

Then,

$$\|u^* - u^{\text{so}}\| \leq \frac{2\ell\|G - \hat{G}\|}{\mu\sigma^2_{\min}(\hat{G})}$$

Stable optimizer and tracking bound

Theorem: Under the same assumptions as previous theorem

Define: $e_k^{\text{so}} := \| \begin{bmatrix} u_k \\ x_k \end{bmatrix} - \begin{bmatrix} u_k^{\text{so}} \\ x_k^{\text{so}} \end{bmatrix} \|$ $\qquad \text{samp-err}_k := \nabla_1\phi(u_k, y_k) + G^\mathsf{T}\nabla_2\phi(u_k, y_k) - \mathbb{E}_{y_k}[\nabla_1\phi(u_k, y_k) + G^\mathsf{T}\nabla_2\phi(u_k, y_k)]$

Then, for any $k \geq 0$:

$$\mathbb{E}[e_{k+1}^{\text{so}}] \leq \beta\mathbb{E}\left[e_k^{\text{so}}\right] + \gamma\mathbb{E}[\sup_{0\leq\tau\leq k} \|e_{\tau+1}^{\text{so}} - e_\tau^{\text{so}}\|] + \eta\mathbb{E}[\|\text{samp-err}_k\|]$$

Where $\beta = \max\{\boxed{\sqrt{1-\eta\mu} + \eta\hat{\ell}^\nabla\|G - \hat{G}\|}, \sqrt{\frac{\bar{\lambda}(P)}{\underline{\lambda}(P)}\left(1 - \frac{(1-\kappa)\underline{\lambda}(Q)}{\bar{\lambda}(P)}\right)} + \eta\hat{\ell}^\nabla\|C\|\}$

Impose < 1

Implications:

- Tracking error is contractive if $\quad \eta < \frac{1}{\mu} \quad$ and $\quad \|G - \hat{G}\| \leq \frac{\mu}{\hat{\ell}^\nabla}$
- Error bound depends on **temporal variability of optimizer** and **sampling error**

Continuous-time implementations and constrained optimization

Continuous-time implementations

System to control:

$$\dot{x} = Ax + Bu + Ew_t$$
$$y = Cx + Dw_t$$

Noise model:

w_t deterministic & time-varying

Steady-state map:

$$y = \underbrace{-CA^{-1}B}_{G}\, u + \underbrace{(D - CA^{-1}E)}_{H}\, w$$

Control objective:

$$\min_{u^d} \quad \phi_t(u^d, Gu^d + Hw_t)$$

Define measurements-based gradient:

$$\Phi_t^s(u, y) = \nabla_1\phi_t(u, y) + G^\mathsf{T}\nabla_2\phi_t(u, y)$$

Online gradient flow:

$$\dot{u} = -\eta\Phi_t^s(u, y) \qquad \begin{aligned} \dot{x} &= Ax + Bu + Ew_t \\ y &= Cx + Dw_t \end{aligned}$$

For time-invariant optimization problems

A. Hauswirth, S. Bolognani, G. Hug, and F. Dörfler, "Timescale separation in autonomous optimization," IEEE Transactions on Automatic Control, vol. 66, no. 2, pp. 611–624, 2021.

Projected gradient-flows

System to control:

$$\dot{x} = Ax + Bu + Ew_t$$
$$y = Cx + Dw_t$$

Noise model:

w_t deterministic & time-varying

Steady-state map:

$$y = \underbrace{-CA^{-1}B}_{G}\,u + \underbrace{(D - CA^{-1}E)}_{H}\,w$$

Control objective:

$$\min_{u^d} \quad \phi_t(u^d, Gu^d + Hw_t)$$
$$\text{s.t.} \quad u^d \in \mathcal{U}$$

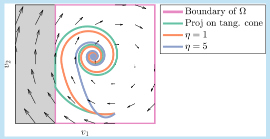
Define measurements-based gradient:

$\Phi_t^{\mathrm{s}}(u, y) = \nabla_1 \phi_t(u, y) + G^\mathsf{T} \nabla_2 \phi_t(u, y)$

Projected gradient flow:

$$\dot{u} = P_{\mathcal{U}}\big(u - \eta \Phi_t^{\mathrm{s}}(u, y)\big) - u \qquad \dot{x} = Ax + Bu + Ew_t$$
$$y = Cx + Dw_t$$

cf. Projection on tangent cone



$$\dot{v} = \lim_{\delta \to 0^+} \frac{P_\Omega(v + \delta F(v)) - v}{\delta}$$

Projected primal-dual gradient flows

System to control:

$$\dot{x} = Ax + Bu + Ew_t$$
$$y = Cx + Dw_t$$

Noise model:

w_t deterministic & time-varying

Steady-state map:

$$y = \underbrace{-CA^{-1}B}_{G}\,u + \underbrace{(D - CA^{-1}E)}_{H}\,w$$

Control objective:

$$\min_{u^d} \quad \phi_t(u^d, Gu^d + Hw_t)$$
$$\text{s.t.} \quad u^d \in \mathcal{U}$$
$$K_t\,(Gu^d + Hw(t)) \le k_t$$

Define measurements-based gradients of Lagrangian:

$$L_t^u(u, y, \lambda) := (\nabla\phi_t)(u, y) + G^\mathsf{T} K_t^\mathsf{T} \lambda \qquad L_t^\lambda(y, \lambda) := K_t y - k_t - \nu\lambda$$

Online projected primal-dual gradient flow:

$$\dot{u} = P_{\mathcal{U}}\big(u - \eta L_t^u(u, y, \lambda)\big) - u \qquad \dot{x} = Ax + Bu + Ew_t$$
$$\dot{\lambda} = P_{\mathcal{C}}\big(\lambda + \eta L_t^\lambda(y, \lambda)\big) - \lambda \qquad y = Cx + Dw_t$$

Closed-loop tracking bound

Theorem: Assume

- **Regularity of ϕ_t**: ℓ-Lipschitz gradient and μ-strongly convex
- **Temporal regularity**: $\nabla\phi_t$, K_t, k_t locally Lipschitz in time, w_t locally absolutely continuous
- **Plant properties**: Controllab. + Observab. + Stability: $\exists P, Q: \; A^\mathsf{T}P + PA = -Q$
- Define: $e_t := \| \begin{bmatrix} u \\ \lambda \\ x \end{bmatrix} - \begin{bmatrix} u_t^* \\ \lambda_t^* \\ x_t^* \end{bmatrix} \|$

If $\eta < \max\{\frac{4\mu}{\ell^2}, c_{A,B,C}\}$

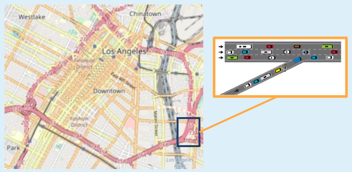
$$e_t \le \underbrace{\beta\|e_0\|\exp(-\alpha t)}_{\text{Effect of initial conditions}} + \underbrace{\gamma_1 \operatorname*{ess\,sup}_{0 \le \tau \le t} \| \begin{bmatrix} \dot{u}_\tau^* \\ \dot{\lambda}_\tau^* \\ \dot{x}_\tau^* \end{bmatrix} \|}_{\text{Shift of optimizer}} + \underbrace{\gamma_2 \operatorname*{ess\,sup}_{0 \le \tau \le t} \|\dot{w}_\tau\|}_{\text{Variability of noise}}$$

Implications:

- If controller gain η is sufficiently small, input-to-state stability bound
- Error bound depends on **temporal variability of noise w_k** and on **shift of optimizer**

Ramp metering

Cell Transmission Model:



$$\dot{x}_i = -f_i^{\text{out}}(x) + f_i^{\text{in}}(x)$$
$$f_i^{\text{out}}(x) = \min\{d_i(x_i), \{s_j(x_j)/r_{ij}\}_{j \in i^+}\}$$
$$d_i(x_i) = \min\{\varphi_i x_i, d_i^{\max}\}$$
$$s_i(x_i) = \min\{\beta_i(x_i^{\text{jam}} - x_i), s_i^{\max}\}$$
$$f_i^{\text{in}}(x) = \sum_{j \in i^-} f_j^{\text{out}}(x)$$

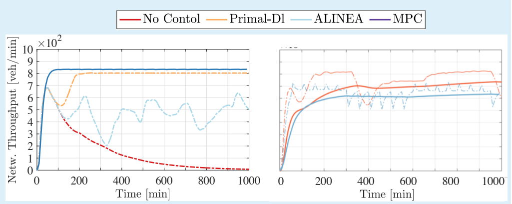
if traffic is controlled in free-flow

$$\dot{x} = Ax + Bu + Ew_t$$
$$y = Cx + Dw_t$$

Steady-state control, constrained to free-flow:

$$\min_{u,y} \quad (u - u^{\text{ref}})^\mathsf{T} Q_u (u - u^{\text{ref}}) - \Phi(y) \qquad \text{(reference tracking)}$$
$$\text{s.t.} \quad y = -((R^\mathsf{T} - I)F)^{-1}Bu + w \qquad \text{(steady-state map)}$$
$$u_i \ge 0, \quad y_i \le \min\{x_i^{\text{crt},d}, x_i^{\text{crt},s}\} \qquad \text{(free-flow traffic)}$$

Test case from Los Angeles, USA



without disturbances **with disturbances**

ALINEA:
Papageorgiou, Kotsialos, "Freeway ramp metering: An overview," *IEEE T-ITS*, 2002

Conclusions

- Increasing **need to combine control and optimization**
  (Benefits: robustness against variable disturbances, simple controller structure)
- **Nontrivial** to "feed" state of optimization as input to physical system
- Analysis reveals controller design strategies
- **Input-to-state stability** type guarantees
- Several extensions (**data-driven, continuous-time, constraints**)

Thank you!

Special thanks to collaborators: