Data-driven Adaptive Event-Triggered Control for Discrete-Time Systems ⋆

Vijayanand Digge * **Akshit Saradagi** ** **Gianluca Bianchin** *

* *The authors are with ICTEAM Institute and the Department of Mathematical Engineering at UCLouvain, Belgium.* {vijayanand.digge, gianluca.bianchin}@uclouvain.be
** *The author is with the Robotics and AI group, in the Department of Computer Science, Electrical and Space Engineering at Lulea University of Technology, Sweden,* akshit.saradagi@ltu.se

Abstract: This paper presents a data-driven framework for synthesizing adaptive event-triggered control strategies for discrete-time linear systems with unknown dynamics. A key feature of the proposed method is the design of an adaptive (state-dependent) relative threshold and absolute threshold-based triggering mechanism, meaning that the triggering threshold is not fixed, but instead adapts online based on the current system state at each event time. The controller gain and the associated adaptive triggering rule are obtained by solving a set of data-dependent optimization problems expressed as linear matrix inequalities. This synthesis procedure provides closed-loop guarantees such as exponential stability and desired performance levels while relying exclusively on collected data rather than an explicit model of the system. Simulation results on illustrative examples validate the proposed methodology. In particular, they demonstrate that the thresholding strategy, by adapting to the instantaneous state of the system, yields substantially longer inter-event times when compared to classical static (constant) triggering rules. This leads to significant reductions in communication load. These results highlight the potential of adaptive event-triggered control schemes as an effective tool for resource-aware control of modern networked systems.

Keywords: Event-based control, Data-based control, Linear systems

1. INTRODUCTION

Event-triggered control (ETC) has emerged as a powerful framework within networked and resource-constrained control systems. Its primary goal is to improve the efficiency of limited resources such as communication bandwidth by transmitting measurements or updating control inputs only when certain conditions are violated, rather than at fixed periodic sampling instants. This makes ETC particularly attractive in applications where communication costs must be minimized, including multi-agent coordination (Li et al. (2023)), cyber-physical systems, and other networked environments (Heemels et al. (2012)).

In a typical ETC scheme, a triggering condition is evaluated at each sampling instant; once violated, a new measurement is sent to the controller, which updates its input accordingly. Between two triggering instants, the control input is held constant. Several triggering mechanisms have been proposed in the literature. A relative threshold-based ETC scheme was introduced in Tabuada (2007), while an absolute thresholding strategy was presented in Heemels et al. (2008) for continuous-time systems. Most classical ETC approaches rely on static triggering rules, where the threshold remains constant over time. More recently, dynamic event-triggered mechanisms (DETMs) have been introduced as a way to further reduce communication while ensuring stability (Ge et al. (2021)). Authors in Girard (2014) employ an auxiliary internal variable evolving according to its own dynamics to shape the triggering behavior.

In contrast to DETMs, which rely on the evolution of an internal dynamic variable, another class of approaches computes the threshold as an explicit function of the state measured at the event time, resulting in a adaptive (or state-dependent) threshold. By state-dependent, we mean that the ETC parameter determining whether a new transmission is required is not fixed; instead, it adapts online based on the event state itself, without relying on any additional internal dynamics or memory. This concept was explored in Saradagi et al. (2019) for continuous-time linear systems, where a state-dependent thresholding mechanism was shown to reduce controller transmissions significantly while maintaining stability guarantees.

Despite the extensive literature on ETC, the majority of existing methods are fundamentally model-based. Both controller and trigger designs typically require explicit knowledge of the system matrices. However, in many practical scenarios, first-principle modeling may be difficult, unreliable, or too costly, motivating the need for data-driven control. Data-driven methods aim to learn controllers directly from measured data, without relying on system identification. Recent results have established

⋆ This work was supported by ARC via the SIDDARTA project, UCLouvain, Belgium.

data-driven formulations for stabilization, optimal control, and robust control of linear systems (De Persis and Tesi (2019); Van Waarde et al. (2020)). There has been growing interest in integrating ETC with data-driven methodologies. For discrete-time systems, data-driven ETC approaches have been investigated in Digge and Pasumarthy (2022); Wang et al. (2023), while continuous-time counterparts are presented in De Persis et al. (2023); Wei et al. (2023). More recently, Xu et al. (2024) introduced a data-driven dynamic ETC scheme, reflecting the increasing attention being given to combining data-driven control techniques with event-triggered implementations. Data-driven ETC offers distinct advantages: triggering and control decisions are made directly from measured data, providing greater flexibility and adaptability than model-based strategies (De Persis et al. (2023)). This naturally leads to both controller synthesis and triggering-rule designs as data-dependent optimization problems, frequently expressed through data-driven linear matrix inequalities (LMIs).

These developments motivate the present work, which focuses on data-driven control of discrete-time linear systems operating under event-triggered strategies. Our main contribution is the design of adaptive relative thresholding and adaptive absolute thresholding ETC schemes that are synthesized directly from data, without requiring model identification. This extends the continuous-time, model-based ETC framework of Saradagi et al. (2019) to the discrete-time setting and makes it fully data-driven. Moreover, unlike Saradagi et al. (2019), our discrete-time formulation does not evaluate the triggering condition at every sampling instant; instead, the threshold is recomputed only at event times, thereby reducing online computational effort.

The proposed framework also advances the data-driven ETC methodology of Digge and Pasumarthy (2022). While Digge and Pasumarthy (2022) considers only static relative thresholding, we develop both adaptive relative thresholding and adaptive absolute thresholding within a unified, data-driven LMI-based synthesis procedure. The resulting triggering conditions rely solely on data-deriven matrices and can be computed through convex optimization, avoiding iterative techniques such as the Dinkelbach algorithm employed in Saradagi et al. (2019).

The remainder of the paper is organized as follows. Section 2 presents the data-driven ETC problem formulation. Preliminaries are presented in Section 3. Section 4 develops the proposed data-driven adaptive thresholding-based ETC. Simulation results are provided in Section 5. Concluding remarks are given in Section 6.

Notation: we denote by $\mathbb{R}$, the set of real numbers, and $\mathbb{Z}_{\geq 0}$ the set of non-negative integers. The Euclidean vector norm is represented by $\|\cdot\|$. I refers to the identity matrix with an appropriate dimension. $\mathbb{S}^n$ denotes the set of all $n \times n$ real symmetric matrices. Given a symmetric matrix, the notation $M \succ 0 (M \succeq 0)$ and $M \prec 0 (M \preceq 0)$ means that M is a positive and negative (semi)- definite matrix.

2. PROBLEM FORMULATION

Consider the discrete-time linear system:

$$x(k+1) = Ax(k) + Bu(k), \tag{1}$$

where $k \in \mathbb{Z}_{\geq 0}$ denotes time, $x(k) \in \mathbb{R}^n$ is the state, and $u(k) \in \mathbb{R}^m$ is the input. Matrices A and B are real, constant, unknown and assumed to be such that $(A,\, B)$ is stabilizable. In this work, we consider networked control scenarios in which the plant and controller exchange information over a communication network, where communication may occur intermittently. Between two events, following a sample-and-hold implementation, the controller applies a constant input based on the most recent transmitted state:

$$u(k) = Kx(k_i), \qquad k \in [k_i,\, k_{i+1}), \tag{2}$$

where $k_i, k_{i+1} \in \mathbb{Z}_{\geq 0}$ denote consecutive communication instants. The asynchronous nature of the control updates induces a state measurement error defined as

$$e(k) = x(k_i) - x(k), \tag{3}$$

where $x(k_i)$ is the state at the most recent control update and $x(k)$ is the current state. In this work, we focus on two classes of triggering mechanisms that determine the communication instants, described next.

Relative thresholding: Motivated by ETC architectures, we first consider the *relative-thresholding* triggering rule (see Goebel et al. (2012)), defined as

$$\|e(k)\| \leq \sigma \|x(k)\|, \tag{4}$$

where $\sigma > 0$ is a fixed design parameter. Whenever (4) is violated, a new state measurement is transmitted and the control input is updated. The rule in (4) is *static*, in the sense that a single constant threshold σ is selected and applied throughout the entire closed-loop operation.

Although simple to implement, the condition (4) can be overly conservative. In particular, σ must be chosen to guarantee stability *for all possible states*, which typically forces its value to reflect a worst-case scenario. To mitigate this conservatism, we consider a more flexible mechanism based on an *adaptive relative-thresholding* rule:

$$\|e(k)\| \leq \sigma_k \|x(k)\|, \tag{5}$$

where the parameter $\sigma_k > 0$ is allowed to vary with time.

Absolute Thresholding: We next consider an *adaptive absolute-thresholding* rule of the form

$$\|e(k)\| \leq \varepsilon_k, \tag{6}$$

where $\varepsilon_k \geq 0$ is a time-varying threshold. Analogously to the mechanism in (4), whenever the measurement error exceeds ε_k, a new state measurement is transmitted and the controller is updated.

The problem of interest is formulated next.

Problem 1. Suppose that $K \in \mathbb{R}^{m \times n}$ is such that the control law $u(k) = Kx(k)$ stabilizes the system (1). Design an adaptive relative-thresholding rule (cf. (5)) and an adaptive absolute-thresholding rule (cf. (6)) that maximize sampling efficiency while ensuring closed-loop stability. This design must rely solely on finite-length state and input trajectories $\{x(k)\}_{k=0}^{T}$ and $\{u(k)\}_{k=0}^{T-1}$ obtained from an open-loop experiment, without requiring any knowledge of the system matrices A and B. □

3. PRELIMINARIES

We introduce the preliminaries used throughout.

Definition 3.1. (Exponential stability). A discrete-time linear system $x(k+1) = Ax(k)$, $A \in \mathbb{R}^n$ is exponentially stable if there exists a function $V : \mathbb{R}^n \to \mathbb{R}$ defined by $V(x(k)) = x(k)^\top P x(k)$, $P = P^\top \succ 0$, such that $V(x(k+1)) \leq \lambda V(x(k))$ along the trajectories of the system for all $k \geq 0$ and for some $\lambda \in (0\ 1)$.

3.1 S-procedure

Our work relies on the S-procedure (Boyd and Vandenberghe (2004)), stated below for completeness. Let $A_1, A_2 \in \mathbb{S}^n$, $b_1, b_2 \in \mathbb{R}^n$. and $c_1, c_2 \in \mathbb{R}$, The implication

$$x^\top A_1 x + 2b_1^\top x + c_1 \leq 0 \implies x^\top A_2 x + 2b_2^\top x + c_2 \leq 0,$$

holds if there exists a multiplier $\mu \in \mathbb{R}$ such that

$$\mu \geq 0, \begin{bmatrix} A_2 & b_2 \\ b_2^\top & c_2 \end{bmatrix} \preceq \mu \begin{bmatrix} A_1 & b_1 \\ b_1^\top & c_1 \end{bmatrix}.$$

3.2 Persistence of Excitation

Consider the discrete-time linear system (1). Let $\{x(k)\}_{k=0}^{k=T}$ and $\{u(k)\}_{k=0}^{k=T-1}$ denote sequences of state and input measurements collected from an open-loop experiment. We arrange this data into matrices:

$$\begin{aligned} U &= [u(0)\ u(1)\ \dots\ u(T-1)], \\ X_- &= [x(0)\ x(1)\ \dots\ x(T-1)], \\ X_+ &= [x(1)\ x(2)\ \dots\ x(T)]. \end{aligned}$$

Suppose that the input sequence $\{u(k)\}_{k=0}^{T-1}$ is persistently exciting (Willems et al. (2005)); then, De Persis and Tesi (2019) showed that

$$\text{rank} \begin{bmatrix} X_- \\ U \end{bmatrix} = n + m. \tag{7}$$

Note that $T > (m+1)n + m$ is necessary for persistence of excitation to hold. The rank condition (7) implies that the whole set of trajectories of a linear system can be represented by a finite set of system trajectories.

3.3 Data-driven representation

The closed-loop system with control input of the form (2) and using (3) is

$$x(k+1) = (A + BK)x(k) + BKe(k), \quad k \in [k_i, k_{i+1}) \tag{8}$$

which can be viewed as the closed-loop system subject to the state measurement error $e(k)$. Since A and B are unknown, we use data-driven representation of the closed-loop system (8) shown in Digge and Pasumarthy (2022).

Lemma 1. (Digge and Pasumarthy (2022)) Let the rank condition (7) hold. Then, system (8) has the equivalent data-driven representation

$$x(k+1) = X_+ G x(k) + X_+ H e(k) \tag{9}$$

where G and H are any solution to the system of equations

$$\begin{bmatrix} I & 0 \\ K & K \end{bmatrix} = \begin{bmatrix} X_- \\ U \end{bmatrix} [G\ H] \tag{10}$$

which exists under the assumption (7).

3.4 Data-driven Controller Design

Let $u(k) = Kx(k)$. Then, (1) can be rewritten as (see De Persis and Tesi (2019)):

$$x(k+1) = X_+ G x(k) \tag{11}$$

Lemma 2. (De Persis and Tesi (2019); Digge and Pasumarthy (2022)) Let the rank condition (7) hold. A state-feedback gain that exponentially stabilizes the system (11) with rate $\lambda \in (0, 1)$ as in Definition 3.1 is $K = UQ_1(X_-Q_1)^{-1}$ where $X_-Q_1 \succ 0$ with Q_1 satisfying:

$$\begin{bmatrix} \lambda X_- Q_1 & Q_1^\top X_+^\top \\ X_+ Q_1 & X_- Q_1 \end{bmatrix} \succeq 0. \tag{12}$$

Lemma 2 provides a way to design the state feedback controller gain K in the absence of a network. Note that $Q_1 = GP^{-1}$, and G as in (10) with a change-of-variables used to obtain a convex feasibility problem.

4. ADAPTIVE EVENT-TRIGGERED CONTROL

In this section, we address Problem 1. The exposition is organized into two parts: we first develop an adaptive relative-thresholding mechanism, and subsequently derive the corresponding adaptive absolute-thresholding rule.

4.1 Relative Thresholding

We begin with a technical reformulation of the problem. By Lemma 2, any stabilizing controller for (1) can be written as:

$$K = UQ_1(X_-Q_1)^{-1} \tag{13}$$

where X_-Q_1 is positive definite for some Q_1 satisfying (12). Consider the candidate Lyapunov function:

$$V(k) = x(k)^\top P x(k), \quad P \succ 0,$$

where $P = (X_-Q_1)^{-1}$. Then, given $\lambda \in (0\ 1)$, direct substitution gives:

$$V(x(k+1)) - \lambda V(x(k)) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}^\top \begin{bmatrix} -Q & S \\ S^\top & R \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix}, \tag{14}$$

where

$$\begin{aligned} Q &= \lambda P - G^\top X_+^\top P X_+ G, \\ S &= G^\top X_+^\top P X_+ H, \\ R &= H^\top X_+^\top P X_+ H, \end{aligned}$$

and G and H are any solution to (10). Notice that all these matrices can be computed solely from data. By substituting (3) into (14) and by setting $V(x(k+1)) - \lambda V(x(k)) \leq 0$, we obtain:

$$\begin{bmatrix} x(k) \\ 1 \end{bmatrix}^\top \underbrace{\begin{bmatrix} -Q - (S + S^\top) + R & (S - R)x(k_i) \\ x(k_i)^\top (S^\top - R) & x(k_i)^\top R x(k_i) \end{bmatrix}}_{M_d(x(k_i))} \begin{bmatrix} x(k) \\ 1 \end{bmatrix} \leq 0. \tag{15}$$

Next, we note that the inequality (5) using (3) can be recast as:

$$\begin{bmatrix} x(k) \\ 1 \end{bmatrix}^\top \underbrace{\begin{bmatrix} (1 - \sigma_k^2)I & -x(k_i) \\ -x(k_i)^\top & x(k_i)^\top x(k_i) \end{bmatrix}}_{RT(\sigma_k, x(k_i))} \begin{bmatrix} x(k) \\ 1 \end{bmatrix} \leq 0. \tag{16}$$

It follows that global exponential stability of the closed-loop system (9) is ensured provided that the following implication holds:

$$z(k)^\top RT(\sigma_k, x(k_i))z(k) \le 0 \implies z(k)^\top M_d(x(k_i))z(k) \le 0, \tag{17}$$

where $z(k) = \begin{bmatrix} x(k)^\top & 1 \end{bmatrix}^\top$.

By the S-procedure (cf. Section 3.1), the implication (17) holds provided that there exists a scalar $\mu \ge 0$ such that:

$$\mu RT(\sigma_k, x(k_i)) - M_d(x(k_i)) \succeq 0. \tag{18}$$

The next proposition formulates the optimization problem to select σ_k to ensure the desired stability property.

Proposition 1. Suppose that the rank condition (7) holds, and let K be defined as in (13). For a given state $x(k_i)$, consider the optimization problem

$$\begin{aligned} \max_{y_k,\, t>0} \quad & y_k \\ \text{s.t.} \quad & RT(\sigma_k, x(k_i)) - t\, M_d(x(k_i)) \succeq 0, \end{aligned} \tag{19}$$

where the matrices $M_d(x(k_i))$ and $RT(\sigma_k, x(k_i))$ are defined in (15) and (16), respectively. Let (y_k, t) denote an optimal solution to (19). Then the closed-loop system (1), under the control law (2) and the adaptive relative-thresholding rule (5) with $\sigma_k^2 = y_k$, is globally exponentially stable.

Proof. We first show that the corresponding optimization problem has a nonempty feasible set; it is sufficient to exhibit one feasible point. We consider the quadratic form $z^\top\big(RT_0 - tM_d\big)z$, where $RT_0 = RT(0, x(k_i))$ for simplicity, for $z = \begin{bmatrix} x(k)^\top & 1 \end{bmatrix}^\top$, and distinguish two cases depending on whether $x(k)$ coincides with the last transmitted state $x(k_i)$.

Case 1 ($z \in \ker(RT_0)$ i.e., $x(k) = x(k_i)$). Define $v := \begin{bmatrix} x(k_i)^\top & 1 \end{bmatrix}^\top$. A direct computation shows that $RT_0 v = 0$, so v belongs to the nullspace of RT_0. From the Lyapunov condition (see (15)), we have $v^\top M_d v \le 0$. Therefore,

$$v^\top\big(RT_0 - tM_d\big)v = -t\, v^\top M_d v \ \ge\ 0 \quad \text{for all } t \ge 0,$$

that is, the LMI inequality holds automatically along the direction corresponding to $x(k) = x(k_i)$.

Case 2 ($z \notin \ker(RT_0)$ i.e., $x(k) \ne x(k_i)$): In this case, the vector z is not a multiple of v, from which it follows

$$z^\top RT_0 z = \|x(k) - x(k_i)\|^2 > 0.$$

We can lower bound

$$z^\top RT_0 z \ge \underline{\sigma}(RT_0)\|z\|^2, \quad z \notin \ker(RT_0), \tag{20}$$

where $\underline{\sigma}$ is minimum non-zero eigenvalue of RT_0. Using the bound (20) and the Cauchy-Schwarz bound, we have

$$z^\top\big(RT_0 - tM_d\big)z \ \ge\ \underline{\sigma}(RT_0)\|z\|^2 \ -\ t\,\|M_d\|\,\|z\|^2,$$

Therefore, for any z with $x(k) \ne x(k_i)$,

$$z^\top\big(RT_0 - tM_d\big)z \ \ge\ 0 \quad \text{whenever} \quad t \le t^\star,$$

with $t^\star := \frac{\underline{\sigma}(RT_0)}{\|M_d\|}$.

Thus, merging both cases, we can choose a strictly positive lower bound t_0 such that $0 < t_0 \le t^\star$ for all z which shows that the feasible set of (19) is nonempty.

Finally, observe that the S-procedure condition (18) is equivalent to the LMI in (19) by multiplying both sides by $1/\mu, (\mu > 0)$ and setting $t = 1/\mu$. Therefore, since (18) guarantees $V(x(k+1)) \le \lambda V(x(k))$ whenever the triggering condition holds, and the controller from Lemma 2 ensures the same at event instants, global exponential stability follows. ■

Algorithm 1 Adaptive Event-Triggered Control

Input: Data matrices U, X_-, X_+ such that (7) holds.
Obtain state-feedback controller (13) with rate λ.
Compute the Lyapunov matrices P, Q, S, R
Set $x(k_0) = x(0)$ (First event sample)
Compute initial threshold σ_0 using optimization problem (19) at $x(k_0)$
Set $k \leftarrow 1$
while $k < k_{\max}$ **do**
    Compute error $e(k) = x(k_i) - x(k)$
    if $\|e(k)\| < \sigma_k\|x(k)\|$ **then** (No event)
        Apply $u(k) = Kx(k_i)$
    else (Event triggered)
        $x(k_i) \leftarrow x(k)$
        Apply $u(k) = Kx(k_i)$
        Recompute threshold σ_{k+1} using optimization problem (19)
    end if
    $k \leftarrow k + 1$
end while

The overall adaptive relative thresholding-based ETC procedure is summarized in Algorithm 1, which describes the online computation of the state-dependent threshold parameter.

Remark 4.1. The proposed adaptive ETC shifts computational effort from the time domain to the event domain, potentially reducing the frequency of optimization calls when the system state evolves benignly.

4.2 Absolute Thresholding

We now consider an alternative triggering strategy based on absolute thresholding ETC. In the static absolute triggering condition, the $\varepsilon > 0$ for all $k \ge 0$ ($\varepsilon = 0$ corresponds to trivial triggering, meaning an event is triggered at every time step), remains constant as defined in (6). This inequality can be written as the quadratic constraint

$$\begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix}^\top \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & I_n & 0 \\ 0 & 0 & -\varepsilon^2 \end{bmatrix}}_{AT(\varepsilon)} \begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix} \le 0. \tag{21}$$

which is equivalent to $\|e(k)\|^2 - \varepsilon^2 \le 0$. Under the Lyapunov function (14), the closed-loop stability requirement can be written as

$$\begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix}^\top \underbrace{\begin{bmatrix} -Q & S & 0 \\ S^\top & R & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{V} \begin{bmatrix} x(k) \\ e(k) \\ 1 \end{bmatrix} \le 0, \tag{22}$$

To certify stability, we require the triggering condition (21) to imply (22). Using the S-procedure introduced in Section 3.1, such an implication holds if there exists a scalar $\mu \ge 0$ satisfying

$$\mu AT(\varepsilon) - V \succeq 0$$

We can see that this resultant matrix inequality will never be positive semi-definite. Due to the presence of $-\mu\varepsilon^2$ in bottom-right entry of $\mu AT(\varepsilon)-V$. Which makes $-\mu\varepsilon^2 < 0$ for any $\mu > 0$ and if $\mu = 0$, the condition $\mu AT(\varepsilon) - V \succeq 0$ reduces to $-V \succeq 0$, which is not generally true. Hence, static absolute thresholding cannot be certified via the S-procedure. To overcome this limitation, we replace the constant threshold ε with a state-dependent term based on the last transmitted state $x(k_i)$.

We use $\|x(k_i) - x(k)\|^2 \leq \varepsilon_k^2$ from definition of measurement error (3), which is equivalent to the quadratic constraint

$$\begin{bmatrix} x(k) \\ 1 \end{bmatrix}^\top \underbrace{\begin{bmatrix} I & -x(k_i) \\ -x(k_i)^\top & -\varepsilon_k^2 + x(k_i)^\top x(k_i) \end{bmatrix}}_{AT(\varepsilon_k, x(k_i))} \begin{bmatrix} x(k) \\ 1 \end{bmatrix} \leq 0. \quad (23)$$

To certify stability, we require the triggering condition (23) to imply (15). Using the S-procedure described in Section 3.1 the implication holds provided that there exists a scalar $\mu \geq 0$ such that:

$$\mu AT(\varepsilon_k, x(k_i)) - M_d(x(k_i)) \succeq 0, \quad (24)$$

which is solvable because the state-dependent matrix introduces a positive term $x(k_i)^\top x(k_i)$ in the lower-right entry of the resultant quadratic inequality.

Proposition 2. Suppose that the rank condition (7) holds, and let K be defined as in (13). For a given state $x(k_i)$, consider the optimization problem

$$\max_{y_k,\, t>0} \quad y_k$$
$$\text{s.t.} \quad AT(\varepsilon_k, x(k_i)) - t\, M_d(x(k_i)) \succeq 0, \quad (25)$$

$AT(\varepsilon_k, x(k_i)), M_d(x(k_i))$ are defined as in (15) and (23) respectively. Let (y_k, t) denote an optimal solution to (19). Then the closed-loop system (1), under the control law (2) and the adaptive absolute-thresholding rule (6) with $\varepsilon_k^2 = y_k$, is globally exponentially stable.

Proof. We first show that the corresponding optimization problem has a nonempty feasible set; it is sufficient to exhibit one feasible point (ε_k, t). Observe that for $\varepsilon_k = 0$,

$$AT(0, x(k_i)) = \begin{bmatrix} I & -x(k_i) \\ -x(k_i)^\top & x(k_i)^\top x(k_i) \end{bmatrix} = RT(0, x(k_i)),$$

i.e., the absolute-threshold matrix coincides with the relative-threshold matrix at $\sigma_k = 0$. From the feasibility analysis of the relative thresholding case in Proposition 1, there exists $t > 0$ such that

$$RT(0, x(k_i)) - tM_d(x(k_i)) \succeq 0.$$

Therefore, the same t together with $\varepsilon_k = 0$ satisfies

$$AT(0, x(k_i)) - tM_d(x(k_i)) \succeq 0,$$

which shows that the feasible set of (25) is nonempty.

Finally, note that the S-procedure condition (24) is equivalent to the LMI in (25) via the change of variables $t = 1/\mu$ and rescaling both sides by $1/\mu$. Since (24) guarantees $V(x(k+1)) \leq \lambda V(x(k))$ whenever the triggering condition holds, and the controller from Lemma 2 ensures the same at event instants, global exponential stability follows. ■

Implementation proceeds identically to Algorithm 1, with the only differences being the triggering condition (6) and the feasibility problem (19) replaced by (25).

Remark 4.2. The adaptive absolute thresholding mechanism derived above provides a practical way to implement absolute event-triggering while preserving closed-loop stability. A natural extension of this idea is to combine absolute and relative error thresholds into a mixed-triggering strategy of the form $\|e(k)\| \leq \sigma_k \|x(k)\| + \varepsilon_k$. Such hybrid mechanisms allow the designer to exploit the complementary advantages of both schemes. One can tune the trade-off between transient responsiveness and steady-state communication efficiency, achieving improved overall performance in many applications.

5. SIMULATIONS

In this section, we validate results with the numerical example considered in Saradagi et al. (2019); Tabuada (2007). A continuous-time system is discretized using a sampling period $T = 0.05s$, resulting in the discrete-time model of the form (1) with

$$A = \begin{bmatrix} 0.9974 & 0.0539 \\ -0.1078 & 1.1591 \end{bmatrix}, \quad B = \begin{bmatrix} 0.0013 \\ 0.0539 \end{bmatrix}.$$

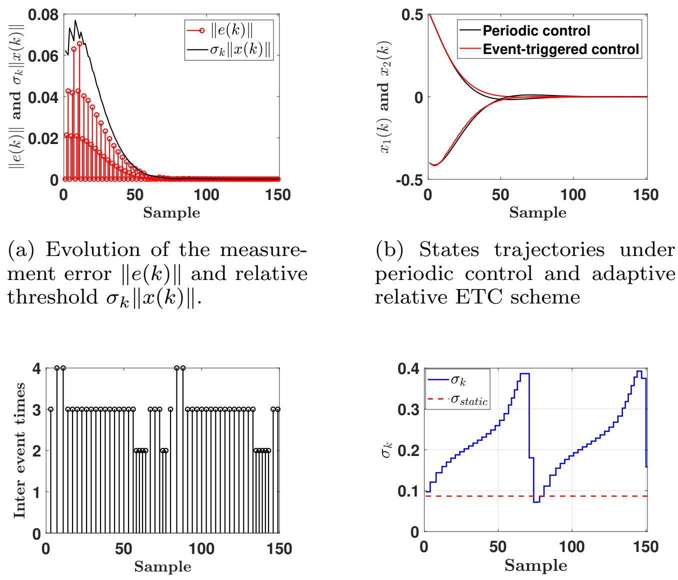
The Eigenvalues of the open-loop system are (1.1053 1.1052), confirming that the system is unstable without feedback. To construct the controller from data, we excite the system with a uniformly distributed random input sequence of length $T = 20$, with initial condition $x(0) = [0.5\ -0.4]^\top$. The collected input-state data satisfy the persistent excitation requirement, and the rank condition (7) holds. The state feedback gain K from (13) that exponentially stabilizes the system with decay rate $\lambda = 0.99$ is found to be $K = [0.1657\ -4.9186]$, which guarantees exponential stability for the ideal (non-networked) closed-loop system.

5.1 Adaptive Relative Thresholding ETC

With the controller fixed, we compute the state-dependent relative threshold σ_k at each event time by solving the optimization problem (19). The evolution of σ_k is shown in Fig. 1d. Each value is computed by solving optimization problem using YALMIP (Löfberg (2004)) with MOSEK as the underlying solver. Figure 1a illustrates the evolution of the measurement error norm $\|e(k)\|$ and the corresponding triggering threshold $\sigma_k\|x(k)\|$ The error remains strictly below the threshold until an event occurs, at which point the control input is updated. In Figure 1c, the inter-event times are shown, with the minimum inter-event time found to be two. Figure 1b shows the trajectory of $x_1(k)$ and $x_2(k)$ under both event-triggered and periodic control. The trajectories nearly coincide, indicating that the ETC law preserves the performance of the nominal closed-loop system while requiring significantly fewer control transmissions.

5.2 Adaptive Absolute Thresholding ETC

We now evaluate the performance of the proposed state-dependent absolute thresholding mechanism described in Section 4.2. As before, the controller K is fixed based on the data-driven synthesis procedure, and the threshold ε_k is computed online at each event time by solving the LMI in (25).



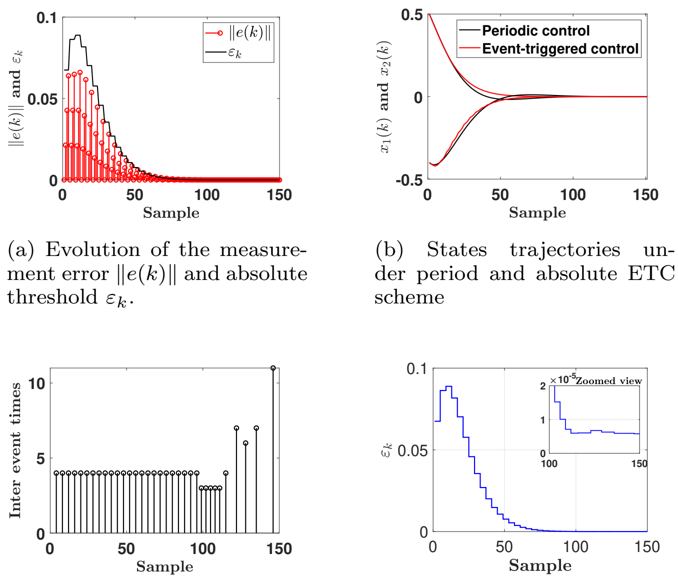
(a) Evolution of the measurement error $\|e(k)\|$ and relative threshold $\sigma_k\|x(k)\|$.

(b) States trajectories under periodic control and adaptive relative ETC scheme

(c) Inter-event times $k_{i+1} - k_i$.

(d) Evolution of σ_k. The static threshold is $\sigma_{\text{static}} = 0.0868$.

Fig. 1. Simulation results of the adaptive relative thresholding event-triggered implementation for $\lambda = 0.99$.



(a) Evolution of the measurement error $\|e(k)\|$ and absolute threshold ε_k.

(b) States trajectories under period and absolute ETC scheme

(c) Inter-event times $k_{i+1} - k_i$.

(d) Evolution of ε_k

Fig. 2. Simulation results of the adaptive absolute thresholding event-triggered implementation for $\lambda = 0.99$.

Figure 2a shows the evolution of the error norm $\|e(k)\|$ together with the adaptive threshold ε_k. As expected, the error remains strictly below the threshold between events, and each threshold update produces a new bound that reflects the magnitude of the last transmitted state $x(k_i)$. The resulting event-triggering behavior is illustrated in Fig. 2c, where the inter-event times remain greater than one. Notably, as the system state converges, the values of ε_k decrease accordingly: this reflects the fact that smaller steady-state errors can be tolerated without violating the Lyapunov condition. Figure 2b compares the state trajectories under periodic control and adaptive absolute ETC. As with the relative thresholding case, the two trajectories closely match, indicating that the proposed mechanism preserves the performance of the nominal stabilizing controller while reducing communication. Finally, Fig. 2d shows the evolution of ε_k over time. The adaptive nature of the threshold is clearly visible: large values of ε_k appear when the state magnitude is large, and the threshold naturally shrinks as the state approaches the origin. This state-dependent scaling is the key factor enabling longer inter-event times without compromising closed-loop stability.

Overall, these results confirm that the adaptive absolute thresholding strategy is a viable and effective alternative to relative thresholding in discrete-time systems.

5.3 Comparison

To assess the benefits of adaptivity and to contrast relative and absolute triggering mechanisms, we compute the static threshold σ using the data-driven relative ETC design proposed in Digge and Pasumarthy (2022), applied to the present numerical example. For this system and data set, the resulting static threshold is $\sigma = 0.0868$, shown as the horizontal curve labelled "σ_{static}" in Fig. 1d. Because this threshold must guarantee stability for all admissible trajectories, it is necessarily conservative. In contrast, the adaptive relative strategy determines σ_k online as a function of the current event state, allowing the triggering bound to vary with the transient dynamics. The same behavior is observed in the adaptive absolute thresholding setting, where the quantity ε_k adapts naturally to the magnitude of the last transmitted state.

Table 1 summarizes the minimum, maximum, and average inter-event times (IETs) for the three triggering mechanisms. Static relative thresholding yields the smallest IETs, reflecting its conservative nature. In contrast, adaptive relative triggering substantially increases the average IET, nearly doubling it compared to the static scheme. The adaptive absolute rule produces the largest IETs, resulting in the sparsest communication pattern among the methods. These results demonstrate that adaptive thresholds significantly reduce conservatism and enable more efficient communication while preserving closed-loop stability within the data-driven control framework.

Table 1. Minimum, maximum, and average IETs for the three ETC schemes. RT - Relative Thresholding; AT - Absolute Thresholding.

Method	min(IETs)	max(IETs)	avg(IETs)
Static RT	1	3	1.31
Adaptive RT	2	4	2.86
Adaptive AT	3	11	4.30

5.4 Computational Performance Analysis

To assess computational efficiency, we benchmarked the optimization problem in (19) on a standard laptop (Intel i7-8650U, 16 GB RAM). Using MATLAB with YALMIP and MOSEK solver, the optimization required on average $0.0236s$ per solve $(\min 0.0142s, \max 0.3749s)$. A Julia implementation using COSMO (Garstka et al. (2021)) achieved faster and more consistent performance, with an average solve time of $0.0156s$ $(\min 0.0090s, \max 0.0160s)$. The substantially lower worst-case execution time of the

Julia–COSMO implementation is particularly promising for real-time use.

6. CONCLUSION

A central contribution of this paper is the introduction of an adaptive thresholding mechanism within a data-driven control framework. Unlike static thresholds, which must be selected conservatively to ensure stability under all operating conditions, the adaptive strategy updates the triggering parameter online based on the current event state. Simulation results demonstrate that this mechanism substantially increases inter-event times compared to static optimal thresholding, thereby achieving significant reductions in communication load.

Future work will investigate robustness in the presence of measurement noise and data imperfections, which are inherent in data-driven control implementations. Another promising direction is the development of computationally efficient solvers or approximate parameterizations that enable real-time deployment of adaptive thresholding with reduced online optimization effort.

REFERENCES

Boyd, S.P. and Vandenberghe, L. (2004). *Convex optimization*. Cambridge university press.

De Persis, C., Postoyan, R., and Tesi, P. (2023). Event-triggered control from data. *IEEE Transactions on Automatic Control*, 69(6), 3780–3795.

De Persis, C. and Tesi, P. (2019). Formulas for data-driven control: Stabilization, optimality, and robustness. *IEEE Transactions on Automatic Control*, 65(3), 909–924.

Digge, V. and Pasumarthy, R. (2022). Data-driven event-triggered control for discrete-time lti systems. In *2022 European Control Conference (ECC)*, 1355–1360. IEEE.

Garstka, J., Sopasakis, P., and Patrinos, P. (2021). Cosmo: A conic operator splitting method for convex conic problems. *Journal of Optimization Theory and Applications*, 190, 779–812.

Ge, X., Han, Q.L., Zhang, X.M., and Ding, D. (2021). Dynamic event-triggered control and estimation: A survey. *International Journal of Automation and Computing*, 18(6), 857–886.

Girard, A. (2014). Dynamic triggering mechanisms for event-triggered control. *IEEE Transactions on Automatic Control*, 60(7), 1992–1997.

Goebel, R., Sanfelice, R.G., and Teel, A.R. (2012). *Hybrid Dynamical Systems: Modeling, Stability, and Robustness*.

Heemels, W., Johansson, K.H., and Tabuada, P. (2012). An introduction to event-triggered and self-triggered control. In *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)*, 3270–3285. IEEE.

Heemels, W., Sandee, J., and Van Den Bosch, P. (2008). Analysis of event-driven controllers for linear systems. *International journal of control*, 81(4), 571–590.

Li, Y., Wang, X., Sun, J., Wang, G., and Chen, J. (2023). Data-driven consensus control of fully distributed event-triggered multi-agent systems. *Science China Information Sciences*, 66(5), 152202. doi:10.1007/s11432-022-3629-1.

Löfberg, J. (2004). Yalmip : A toolbox for modeling and optimization in matlab. In *In Proceedings of the CACSD Conference*. Taipei, Taiwan.

Saradagi, A., Mahindrakar, A.D., and Mohan, S. (2019). Optimization of relative and absolute thresholding parameters in event-triggered control. In *2019 18th European Control Conference (ECC)*, 2737–2742. IEEE.

Tabuada, P. (2007). Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Transactions on Automatic Control*, 52(9), 1680–1685.

Van Waarde, H.J., Eising, J., Trentelman, H.L., and Camlibel, M.K. (2020). Data informativity: a new perspective on data-driven analysis and control. *IEEE Transactions on Automatic Control*.

Wang, X., Berberich, J., Sun, J., Wang, G., Allgöwer, F., and Chen, J. (2023). Model-based and data-driven control of event-and self-triggered discrete-time linear systems. *IEEE Transactions on Cybernetics*, 53(9), 6066–6079.

Wei, Z.J., Xu, C.Y., Liu, K.Z., Qi, W.L., and Sun, X.M. (2023). Data-driven analysis and control of periodic event-triggered continuous-time systems. *International Journal of Robust and Nonlinear Control*, 33(13), 7951–7967.

Willems, J.C., Rapisarda, P., Markovsky, I., and De Moor, B.L. (2005). A note on persistency of excitation. *Systems & Control Letters*, 54(4), 325–329.

Xu, T., Sun, Z., Wen, G., and Duan, Z. (2024). Data-driven dynamic event-triggered control. *IEEE Transactions on Automatic Control*, 69(12), 8804–8811.